A Rheological Constitutive Model for Human Blood via Population Balances

Soham Jariwala, Jeffrey S. Horner, Antony N. Beris, Norman J. Wagner
Center for Research in Soft Matter & Polymers (CRISP), Department of Chemical and Biomolecular Engineering, University of Delaware, Newark, Delaware, USA-19716

Correspondence: sj1@udel.edu

BACKGROUND AND MOTIVATION:

Human blood is a dense suspension of platelets, white blood cells, red blood cells (RBCs) suspended in plasma along with dissolved proteins. Much of its complex rheology at low deformation rates is a result of RBCs forming coin-stack like aggregates called rouleaux.

The kinetics of rouleaux break down and build up due to Brownian motion and shear results in thixotropic behavior. Rouleaux also impart elasticity and yield-stress making human blood a Thixotropic-Elasto-visco-plastic fluid.

Thixotropy Elasticity

viscosity Yield-stress

A more predictive microstructure-rheology relationship could enable us to probe diseases and develop new health diagnostic tools based on changes in the blood rheology.

METHODS:

We use population balances to describe the kinetics of rouleaux formation using particle-based theories.

Key Assumptions:

• RBCs and Rouleaux are approximated as spheres and fractals, respectively.
• Aggregation is based on Smoluchowski kernel.
• Breakage is assumed to be binary and uniform, described using model proposed by Spicer and Pratsinis.
• Dynamic arrest is modeled using a hyperbolic cut-off function.

\[ \beta(\phi) = \tanh(2.66(\phi - \phi_d)/\phi_d) \]

Some advantages of this approach:

• The model uses physiologically relevant parameters such as RBC size and hematocrit
• Physically meaningful parameters like fractal dimension and stability ratio, that can be validated independently can be obtained through fitting the model to experiments.

Microstructure Evolution

Rheological response

Macroscopic stress

Model summary

The model is adapted from coarse-grained population balance model for shear flows developed by Mwsame et al. [1] using method of moments.

Population balance equation describes how aggregate volume fraction (\( \phi_1 \)) changes with time due to Brownian motion and shear. The equation is coarse-grained using monodisperse closure and written in terms of the zeroth moment of the aggregate size distribution (\( \phi_0 \)), or the number density. Highlighted parameters need to be obtained using data fitting.

\[ \frac{d\phi_0}{dt} = 2\beta \frac{k_B T_0}{2\mu W^2\sigma_0^2} \phi_0^2 - 4\alpha_0 \sqrt{\phi_0} \left[ \phi_1^{3/2} - \phi_0 \right] + b_0 \left[ \phi_1^{1.5} - \phi_0 \right] \]

Red blood cells deform at high-shear rates that results in shear thinning that is captured by Cross-like term in viscosity.

\[ \mu(\phi_0, \gamma) = \mu_0 + C_1 \left( \phi_0 - \phi_d \right) \left( \phi_0 - \phi_{\text{max}} \right) \]

Shear stress is based on modified Bingham model with additive elastic and viscous contribution.

RESULTS:

The model was fit to one steady-state and four unidirectional large amplitude oscillatory shear (UD-LAOS) whole blood measurements carried out by Horner et al. [2] for two donors.

Data fitting: Parallel tempering algorithm developed by Armstrong et al. [3] was used to fit the model to rheological data set.

Macroscopic stress

UD-LAOS Pipkin diagram

Model is reasonably accurate at wide range of shear-rates, especially at physiologically relevant ones (highlighted).

CONCLUSION:

We demonstrate that human blood rheology can be accurately modeled and predicted using population balance modeling of the rouleaux using existing colloidal physics models. This research opens the possibility for using rheology as a medical diagnostic tool.

REFERENCES:


ACKNOWLEDGEMENT:

The authors would like to thank the National Science Foundation (NSF) for funding through awards CBET 1510837 and CBET 1804911.