1) P8.7 Ethylene at 350°C and 50 bar is passed through an adiabatic expander to obtain work and exits at 2 bar. If the expander has an efficiency of 80%, how much work is obtained per mole of ethylene, and what is the final temperature of the ethylene? How does the final temperature compare with what would be expected from a reversible expander?

See equations below:

<table>
<thead>
<tr>
<th>T (K)</th>
<th>623.15</th>
<th>V</th>
<th>H</th>
<th>U</th>
<th>S</th>
</tr>
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<tbody>
<tr>
<td>P (MPa)</td>
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<td>J/mol</td>
<td>J/mol</td>
<td>J/molK</td>
</tr>
<tr>
<td>&amp; for 1 root region</td>
<td>9.1954</td>
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</table>

<table>
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<tr>
<th>T (K)</th>
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<th>V</th>
<th>H</th>
<th>U</th>
<th>S</th>
</tr>
</thead>
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<td>P (MPa)</td>
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<td>cm³/mol</td>
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<td>J/mol</td>
<td>J/molK</td>
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<tr>
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<td>9.1954</td>
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<th>U</th>
<th>S</th>
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<td>MPa</td>
<td>J/mol</td>
<td>J/mol</td>
<td>J/molK</td>
</tr>
<tr>
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<td>#NUM!</td>
<td>#NUM!</td>
<td>#NUM!</td>
<td></td>
</tr>
</tbody>
</table>

2) 8.10 Derive the integrated formula for the Helmholtz energy departure for the virial equation (Eqn. 7.7), where B is dependent on temperature only. Express your answer in terms of B and its temperature derivative.

Equation 7.7 given below with other useful expressions.

3) 9.1 The heat of fusion for the ice-water phase transition is 335 kJ/kg at 0°C and 1 bar. The density of water is 1g/cm³ at these conditions and that of ice is 0.915 g/cm³. Develop an expression for the change of the melting temperature of ice as a function of pressure. Quantitatively explain why ice skates slide along the surface of ice for a 100 kg hockey player wearing 10 cm x 01 cm blades. Can it get too cold to ice skate? Would it be possible to ice skate on other materials such as solid CO₂?

See equations below.
\[
\frac{(S - S^i)}{R} = \int_{0}^{\rho} \left[-T \frac{\partial Z}{\partial T} \right] - (Z - 1) \frac{d\rho}{\rho} + \ln Z \quad 8.23
\]

\[
\frac{(H - H^i)}{RT} = \int_{0}^{\rho} \left[rac{\partial Z}{\partial T} \right] \frac{d\rho}{\rho} + Z - 1 \quad 8.24
\]

\[
\frac{(A - A^i)}{RT} = \int_{0}^{\rho} \frac{(Z - 1)}{\rho} d\rho - \ln Z \quad 8.25
\]

\[
\frac{(G - G^i)}{RT} = \int_{0}^{\rho} \frac{(Z - 1)}{\rho} d\rho + (Z - 1) - \ln Z \quad 8.26
\]

Useful formulas at fixed T,V include:

\[
\frac{(A - A^i)}{RT} = \int_{0}^{\rho} \frac{(Z - 1)}{\rho} d\rho \quad 8.27
\]

\[
\frac{(S - S^i)}{R} = \int_{0}^{\rho} \left[-T \frac{\partial Z}{\partial T} \right] - (Z - 1) \frac{d\rho}{\rho} \quad 8.28
\]

\[
\frac{(H - H^i)}{RT} = \int_{0}^{P} \frac{(\partial Z)}{(\partial T)} \frac{dP}{P} \quad 8.29
\]

\[
\frac{(S - S^i)}{R} = \int_{0}^{P} \left[(Z - 1) + T \frac{(\partial Z)}{(\partial T)} \right] \frac{dP}{P} \quad 8.30
\]

\[
A = U - TS = H - PV - TS
\]

\[
\frac{(A - A^i)}{RT} = \frac{(H - H^i)}{RT} - \frac{(S - S^i)}{R} - Z + 1
\]
\[
\frac{dP^{\text{sat}}}{dT} = \frac{\Delta H^{\text{vap}}}{T(V^V - V^L)}
\]

\[
S^V - S^L = \Delta S^{\text{vap}} = \frac{(H^V - H^L)}{T} = \frac{\Delta H^{\text{vap}}}{T}
\]

\[
d\ln P^{\text{sat}} = -\frac{\Delta H^{\text{vap}}}{R(Z^V - Z^L)} d\left(\frac{1}{T}\right)
\]

\[
d\ln P^{\text{sat}} = -\frac{\Delta H^{\text{vap}}}{R} d\left(\frac{1}{T}\right)
\]

\[
Z = 1 + (B^0 + \omega B^1)P_r/T_r \quad \text{or} \quad Z = 1 + BP/RT
\]

where \(B(T) = (B^0 + \omega B^1)RT_c/P_c\)

\[
B^0 = 0.083 - 0.422/T_r^{1.6}
\]

\[
B^1 = 0.139 - 0.172/T_r^{4.2}
\]

Subject to \(T_r > 0.686 + 0.439P_r\) or \(V_r > 2.0\)

\[
R = 8.314 \text{ MPa cm}^3/(\text{mole K}^\circ)
\]
1) P8.7 Ethylene at 350°C and 50 bar is passed through an adiabatic expander to obtain work and exits at 2 bar. If the expander has an efficiency of 80%, how much work is obtained per mole of ethylene, and what is the final temperature of the ethylene? How does the final temperature compare with what would be expected from a reversible expander?

See equations below and PREOS.xlsx outputs.

(P8.7)

\[ T_1 = 623.15 K \]
\[ P_1 = 5 MPa \]

Use PREOS.XLS

\[ S_i = S''_2 \]

<table>
<thead>
<tr>
<th>T (K)</th>
<th>V (cm³/gmol)</th>
<th>H (J/mol)</th>
<th>U (J/mol)</th>
<th>S (J/molK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>623.15</td>
<td>0.986361</td>
<td>1022.041</td>
<td>18750.12</td>
<td>13635.92</td>
</tr>
</tbody>
</table>

⇒ Use Solver in the spreadsheet by changing pressure to 0.2 MPa and fixing the entropy value = 9.1954 J/mol-K and in this case \( S_i = S''_2 = 9.1954 \) J/mol-K then we can find \( \Delta H \) and \( \Delta H' \)

<table>
<thead>
<tr>
<th>T (K)</th>
<th>V (cm³/gmol)</th>
<th>H (J/mol)</th>
<th>U (J/mol)</th>
<th>S (J/molK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>404.71</td>
<td>0.99469</td>
<td>16739.56</td>
<td>6234.912</td>
<td>1887.95</td>
</tr>
</tbody>
</table>

⇒ \( T = 404.71 K \)

\( \Delta H' = 5234.912 - 18750.12 = -13515.21 J/\text{mole} \)

\( \eta = 0.8 \Rightarrow \Delta H = \frac{W}{W} = \frac{H_2 - H_1}{H''_2 - H_1} \)

\( \Rightarrow 0.8 = \frac{H_2 - 18750.12}{5234.912 - 18750.12} \)

\( \Rightarrow H_2 = 7937.95 J/\text{mol} \)

\( \Rightarrow \Delta H = H_2 - H_1 = 7937.95 - 18750.12 \)

\( \Rightarrow \Delta H' = 10812.166 J/\text{mol} \)


<table>
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<tr>
<th>T (K)</th>
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<tr>
<td>452.012672</td>
<td>0.2</td>
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<td>---</td>
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</tr>
</tbody>
</table>

answers for three root region: #NUM! #NUM! #NUM! #NUM!

answers for three root region: #NUM! #NUM! #NUM! #NUM!

⇒ \( T_2 = 452 K \)
2)  

8.10 Derive the integrated formula for the Helmholtz energy departure for the virial equation (Eqn. 7.7), where $B$ is dependent on temperature only. Express your answer in terms of $B$ and its temperature derivative.

Equation 7.7 given below with other useful expressions.

\begin{align*}
(A - A^g) &= \frac{(H - H^g)}{RT} - \frac{(S - S^g)}{R} - Z + 1 \\
\end{align*}

Using the form for $Z = Z(T, P)$:

\[ \int_0^P \frac{(Z - 1)}{P} \, dP - Z + 1 = \frac{BP}{RT} - \frac{BP}{RT} = 0 \]

3)  

9.1 The heat of fusion for the ice-water phase transition is 335 kJ/kg at 0°C and 1 bar. The density of water is 1 g/cm\(^3\) at these conditions and that of ice is 0.915 g/cm\(^3\). Develop an expression for the change of the melting temperature of ice as a function of pressure. Quantitatively explain why ice skates slide along the surface of ice for a 100 kg hockey player wearing 10 cm x 0.01 cm blades. Can it get too cold to ice skate? Would it be possible to ice skate on other materials such as solid CO\(_2\)?

\begin{align*}
\text{SOLUTION:} \\
H_u - H_s &= 335 \text{ J/g (must add heat to melt ice)} \\

\frac{V_i - V_f}{1 \text{g/cm}^3} &= \frac{1}{0.195 \text{g/cm}^3} = -0.0929 \text{ g/cm}^3 \\

\Delta H^m &= \frac{H_u - H_s}{-0.0929 \text{g/cm}^3} \cdot \frac{T[K]}{T(K)} = 335 \text{J/g} \\

\Delta V &= \frac{\Delta T}{T} = \frac{dP}{360 \text{MPa}} \Rightarrow T_i = \frac{\exp \left[ \frac{-(p_i - p)}{360 \text{MPa}} \right]}{T_i} \\

\text{For a 100 kg person with a 10 cm x 0.01 cm blade,} \\

\Rightarrow \Delta T = \frac{9.8 \text{[m/s]}^2 \times 100 \text{[kg]} / 0.001 \text{[cm]} \times 0.0001 \text{[m]}}{980 \times 10^4 \text{Pa} = 98 \text{MPa}} \\

\Rightarrow \Delta T = 273 \times \exp \left[ \frac{-98}{3606} \right] = -266 \text{K} = -7 \text{°C} \\

\text{But if the temperature is lower than -7°C, this person can no longer skate, and might need a} \\
\text{sharper edge on the skates.} \\
\text{Replace ice with CO}_2 \text{ nobody can skate on} \\
\text{since } \Delta P = \hat{V}_i - \hat{V}_f > 0
\end{align*}