Last week the high and low temperature in Fairbanks Alaska were \(-51^\circ F\) and \(-49^\circ F\) (\(-46.1^\circ C\) and \(-45.0^\circ C\)). Is it possible for a person in a parka (a warm coat) to “freeze to death” just due to breathing this cold air? We will assume that you will “freeze to death” if your body temperature reaches \(30^\circ C\). The body reaches hypothermia at a body temperature of \(35.0^\circ C\).

(Assume that the heat production of the body is equal to the heat loss so the body temperature is normally at equilibrium in the absence of breathing.)

Calculate the time it would take to “freeze to death” (to reach \(30^\circ C\)) for a well-insulated person (man) under these conditions. (Assume that the person is a bath of water (77.1 kg) loosing heat to an air stream with an average air flow rate of 620 liter/min, \(T_{\text{in}} = -46^\circ C\), \(T_{\text{out}} = T_{\text{bath}}\), and that the initial water bath temperature is 98.6°F (37.0 °C). Ignore body heat production and heat transfer to the environment. Also calculate the time to reach a state of hypothermia, \(35.0^\circ C\). Assess the importance of ignoring heat production by the body, 2,500 cal/day (7.26 kJ/min).

\[
1000 \text{ L} = 1 \text{ m}^3.
\]

Air density is 1.46897 kg/m\(^3\) at \(-45.5^\circ C\) and 1.007 bar.

Assume \(C_{\text{pair}} = 7/2 \text{ R}\) and \(C_{\text{vair}} = 5/2 \text{ R}\).

The molecular weight of air is 29 g/mole.

For water \(C_p = C_v = 4.184 \text{ J/(g K)}\).

\[R = 8.314 \text{ J/(K mole)}.
\]

1) First write an expression for \(dQ\) from the air stream with \(T_{\text{bath}}\), \((dm_{\text{air}}/dt)\) and \(dt\) as variables.
2) Then write an expression for \(d(m_{\text{bath}} U)\) as a function of \(dT_{\text{bath}}\).
3) Equate the two and collect the \(T_{\text{bath}}\) terms on the left and time on the right.
4) Integrate this expression to get an expression for \(T_{\text{bath}}\) as a function of time.
5) Set \(T_{\text{bath}}\) to \(30^\circ C\) to find the time till “freezing to death”. (Also find for \(35^\circ C\) for hypothermia.)
6) Compare the added \(dQ\) (7.26 kJ/min) \(dt\) with the maximum loss to air \(dQ\) from your expression in item 1 and make an assessment of the impact on the resulting time.
Answers Quiz 3  
CHE 3120  
Jan 26, 2017

1) Air Stream
\[ dQ = C_p \Delta T \left( \frac{dm_{air}}{dt} \right) dt \]
\[ dQ = \frac{7}{2} R \left( T_{in} + 96^\circ C \right) \frac{3.14}{\text{in} \cdot \text{min}} \]
\[ \frac{dm_{air}}{dt} = 620 \frac{\text{lb}}{\text{min}} \times \frac{1 \text{ lb}}{1000 \text{ kg}} = 0.62 \frac{\text{kg}}{\text{min}} \]
\[ \frac{dm_{air}}{dt} = 3.14 \frac{\text{mol}}{\text{min}} \]

\[ dQ = \int 3.14 \frac{J}{\text{mol} \cdot K} \left( T_{in} + 96^\circ C \right) \]

2) \[ dQ = -d(C_{water} \Delta T) \]

\[ = -m_{water} C_{water} \frac{dT_{bulk}}{dt} \]

\[ = -77.100g \times 4.18 \frac{J}{g \cdot K} \times \frac{1}{9} \frac{J}{K} \cdot dT_{bulk} \]

\[ dQ = 322,000 \frac{J}{K} \cdot dT_{bulk} \]

3) \[ \frac{d\frac{T_{bulk}}{K}}{(T_{in} + 96^\circ C)} = -\frac{9.85}{323,000} \frac{1}{K} \cdot dT \]

\[ \frac{dT_{bulk}}{(T_{in} + 96^\circ C)} = -2.84 \times 10^{-3} \frac{1}{K} \cdot dT \]
4) \[ \text{Thallium } ^\circ C \]
\[ \frac{\text{dThallium}}{(\text{Thallium } + 86 ^\circ C)} \]
\[ = \ln \left( \frac{\text{Thallium } ^\circ C + 86 ^\circ C}{83 ^\circ C} \right) \]
\[ x = \left( \frac{\text{Thallium } ^\circ C + 86 ^\circ C}{83 ^\circ C} \right) \]
\[ \text{d}x = \text{dThallium} \]
\[ \ln \left( \frac{\text{Thallium } ^\circ C + 86 ^\circ C}{83 ^\circ C} \right) = -2.89 e^{-3} \text{ min}^{-1} t^2 \]

5) For Thallium = 30 ^\circ C
\[ t = 352 \text{ min} \ln \left( \frac{30 + 86}{83} \right) \]
\[ = 31.0 \text{ min} \]
\[ t = 352 \text{ min} \ln \left( \frac{30 + 86}{83} \right) \]
\[ = 81.60 \text{ min} \]

6) Take maximum \( \Delta T = 83 ^\circ C \)
\[ \frac{\partial Q}{\partial t} = -914 \frac{\text{J}}{\text{K} \text{min}} (83 ^\circ C) \text{ d}t \]
\[ = -75.9 \text{ kJ/min} \text{ d}t \]

we add 7.26 kJ
Feet are about 40% of the heat lost by breathing. You would still have problems. The only thing keeping you alive in Alaska is the slow heat transfer in the body.