**Extinction**

Extinction is the loss of energy out of a beam of radiation as it propagates.

Extinction = absorption + scattering

**Extinction cross section** … analogous to the cross-sectional area of absorbers or scatterers.

- extinction cross section (per particle): \( \sigma_{\text{ext}} \) area [m\(^2\)]
- mass extinction cross section: \( \kappa_{\text{ext}} \) area/mass [m\(^2\)/kg]

**Extinction Coefficient** \( \alpha_e \) [m\(^{-1}\)] … fractional energy removed per unit length.

- extinction cross section x particle number density \( \alpha_e = \sigma_{\text{ext}} N \) m\(^2\) x m\(^{-3}\) = [m\(^{-1}\)]
- mass extinction cross section x density \( \alpha_e = \kappa_{\text{ext}} \rho \) m\(^2\)/kg x kg/m\(^3\) = [m\(^{-1}\)]

Similar notation used for spectral absorption and scattering:
Absorption cross section = \( \sigma_{\text{abs}} \) [m\(^2\)], mass abs. cross section = \( \kappa_{\text{abs}} \) [m\(^2\)/kg], abs. coefficient = \( \alpha_{\text{abs}} \) [m\(^{-1}\)]
Scattering cross section = \( \sigma_{\text{s}} \) [m\(^2\)], scattering coefficient = \( \beta_{\text{s}} \) [m\(^{-1}\)]

**Aerosols**

Aerosols are solid or liquid particles small enough to remain suspended in air. This includes naturally occurring substances such as dust, pollen, dirt, forest-fire smoke, sea salt, etc., and also anthropogenic substances such as smoke, ash, pollutants, etc. Pollutants vary, but include sulfur oxides, nitrogen oxides, sulfuric acid, particulate matter (asbestos fibers, arsenic, iron, copper, nickel, lead…), etc.

Aerosols typically have size comparable to or larger than the wavelength of light, so they behave as "Mie scatterers" (as far as they can be modeled as spheres). Condensation of water molecules causes aerosols to swell in size, greatly decreasing visibility when relative humidity is greater than about 70-80%.
Tri-Modal Aerosol Size Distribution

Aerosols are often separated into three classifications based on particle size:

1) **Aitken particles** (nucleation mode)
   0.001 – 0.1 µm radius

2) **Large particles** (accumulation mode)
   0.1 – 1 µm radius (CCN)

3) **Giant particles** (coarse mode)
   > 1 µm radius

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Aerosol Optical Properties

Aerosols exhibit both absorption (hence emission) and scattering. It is difficult to state "typical" aerosol optical properties because they vary so widely, depending on particle chemistry and size, moisture content of their surrounding air, etc. Here is one set of aerosol measurements from the Tsushima Strait, between Korea and southern Japan.

Vertical profiles of aerosol optical depth (left) and aerosol extinction coefficient (right), measured with an airborne solar radiometer mounted on a Twin Otter aircraft during the ACE Asia experiment in 2001. This is for a case of fairly heavy aerosol, containing industrial pollutants and mineral dust.

Remote Sensing Systems

Aerosol Optical Properties

Vertical profiles of aerosol optical depth and extinction coefficient for light aerosols in the Tsushima Strait between Korea and southern Japan.

The aerosol optical depth falls off at longer wavelengths with a slow spectral dependence of $\lambda^{-0.45}$. The absorption optical depth has a much steeper spectral falloff that follows $\lambda^{-2}$.

Cloud Condensation Nuclei (CCN)

CCN are tiny aerosols that promote cloud growth through a process called “heterogeneous nucleation” (growth of liquid droplets or ice particles through vapor deposition). The size distribution of CCN in continental air is given approximately by the Junge distribution:

\[ n(r) = \frac{b}{R^4} \]  

where \( b \) is a constant that depends on the particle concentration. A typical value is \( b = 2 \times 10^6 \mu m^4/m^3 \).

The quantity \( n(R) \) is a number density, with units of \#/cm^3.

Note: the Junge distribution is valid for particles with radius larger than approximately 0.2 \( \mu m \).
Cloud Optical Properties

Clouds are made up of liquid water droplets or ice crystals. At visible wavelengths, the primary effect of clouds is to greatly increase scattering losses. In the thermal infrared, clouds can have strong absorption and emission. Radio waves generally travel through clouds with relatively little extinction, except at bands such as the 31.6 GHz liquid water absorption band (used to measure cloud liquid water) and at mm-wave frequencies.

Cloud Optics

Many beautiful and interesting optical phenomena can be seen when sunlight or moonlight is scattered by clouds. Here is a sampling...
Cloud Optics

Iridescence at the thin edge of a wave cloud (diffraction by small droplets of liquid water, or possibly small quasi-spherical ice particles).


Cloud Optics

Iridescence in lenticular wave clouds over the Colorado Front Range (tiny ice particles)
Mountain Wave Clouds

Formation of (a) vertically propagating and (b) vertically trapped mountain wave clouds.

Dave Whiteman, PNNL

Vertically Trapped Mountain Wave Clouds

P. J. Neiman
Vertically Propagating Mountain Wave Cloud

Cloud Optics

Cloud Optics

Corona in wave clouds over the Colorado Front Range (tiny ice particles)

Oblong corona in wave clouds. Oblong shape is a result of a particle-size gradient (smaller particles at the top, larger particles at the bottom of the picture).
Cloud Optics

“Asymptotic corona” at a wave cloud edge (liquid water droplets ~ 7.6-16.6 μm dia)

Cloud Optics

Discontinuous corona forming with a mixture of liquid and ice cloud particles (~14-18 μm dia)
Cloud Optics

Iridescence in a thin wave cloud (liquid water droplets)

Cloud Optics

Glory in liquid water clouds
Cloud Optics

Glory in liquid water clouds

Cloud Optics

Parhelion (sundog) in ice clouds
Cloud Optics

22-degree halo in ice clouds
Cloud Optics

Multiple halo in ice clouds over Bozeman, Montana

Cloud Optics

Sunset colors & twilight sky
Radiative Transfer Equation

As a beam of light propagates through a distance \( dz \) in lossy media, its radiance is decreased, through absorption and scattering, according to:

\[
dL_\lambda = -L_\lambda \kappa_{\lambda \rho} dz = -L_\lambda \alpha_\lambda dz
\]

where \( \kappa_{\lambda \rho} \) is the mass extinction cross section [m²/kg], \( \rho \) is density [kg/m³], and \( \alpha_\lambda \) is the extinction coefficient [m⁻¹].

Radiance also may be increased by emission and multiple scattering back into the beam:

\[
dL_\lambda = J_\lambda \kappa_{\lambda \rho} dz = J_\lambda \alpha_\lambda dz
\]

where \( J_\lambda \) is the source function [W/(m² sr µm)].

Combine negative and positive terms:

\[
dL_\lambda = -L_\lambda \kappa_{\lambda \rho} dz + J_\lambda \kappa_{\lambda \rho} dz
\]

Radiative Transfer Equation:

\[
\frac{dL_\lambda}{(\kappa_{\lambda \rho} \rho)} dz = -L_\lambda + J_\lambda
\]

where

- \( \kappa_{\lambda \rho} \) is the mass extinction cross section [m²/kg]
- \( \rho \) is density [kg/m³]
- \( \alpha_\lambda \) is the extinction coefficient [m⁻¹]

Radiative Transfer Equation with Absorption & Emission

In a nonscattering, blackbody medium in local thermodynamic equilibrium, the equation of radiative transfer can be written using the Planck function as the source function:

\[
\frac{dL_\lambda}{(\kappa_{\lambda \rho} \rho)} dz = -L_\lambda + L_{\lambda BB}(T)
\]

Schwarzchild’s equation

If we let the optical thickness be \( \tau_\lambda(z_1, z) = \int_{z_1}^{z} \kappa_{\lambda \rho} dz \), the solution is given by...

\[
L_\lambda(z) = L_\lambda(0) e^{-\tau_\lambda(z_1, 0)} + \int_{0}^{z_1} L_{\lambda BB}(T(z)) e^{-\tau_\lambda(z_1, z)} \kappa_{\lambda \rho} dz
\]

If we know the path profiles of temperature \( T \), density \( \rho \), and absorption coefficient \( \alpha \), we can calculate the radiance at \( z_1 \).
Beer-Bouguer-Lambert Law ("Beer’s Law")

When scattering and emission can be neglected, the radiative transfer equation reduces to…

\[
\frac{dL_\lambda}{(\kappa_\lambda \rho) dz} = -L_\lambda
\]

Integrate over distance \( z_1 \)…

\[
L_\lambda(z_1) = L_\lambda(0) \exp \left( -\int_0^{z_1} (\kappa_\lambda \rho) dz \right)
\]

Beer’s Law: \( L_\lambda(z_1) = -L_\lambda(0) e^{-\kappa_\lambda u} = -L_\lambda(0) e^{-\int_0^{z_1} \kappa_\lambda dz} \)

Beer’s Law can be used to describe propagation in media with weak absorption or scattering (or both), if the appropriate absorption, scattering, or extinction coefficient is used.

Transmissivity for Beer’s Law

When Beer’s Law is valid, we may define the monochromatic transmittance as

\[
T_\lambda = \frac{L_\lambda(z_1)}{L_\lambda(0)} = e^{-\int_0^{z_1} \alpha_\lambda dz}
\]

For a medium with homogeneous absorption, scattering, etc., this can be written without the integral in the exponent, using \( z \) to represent the path length...

\[
T_\lambda = \frac{L_\lambda(z_1)}{L_\lambda(0)} = e^{-\alpha_\lambda z}
\]
**Beer’s Law with Scattering**

For a uniform extinction profile, relatively low irradiance levels, and a reasonably well-collimated beam, the transmission through distance \( z \) is

\[
T = e^{-(\alpha_a + \beta_a + \beta_n)z} = e^{-\alpha_z}z
\]

where \( \alpha_a \) [m\(^{-1}\)] is the absorption coefficient (absorption per unit distance, dependent on atmospheric constituent, number density, and \( \lambda \)),

\( \beta_R \) is the Rayleigh scattering coefficient [m\(^{-1}\)],

\( \beta_a \) is the aerosol (Mie) scattering coefficient [m\(^{-1}\)],

\( \beta_n \) is the nonselective scattering coefficient [m\(^{-1}\)], and

\( \alpha_z \) is the extinction coefficient (absorption + scatter) [m\(^{-1}\)].

Note: \( \beta_a \) and \( \beta_n \) are highly dependent on the number, shape and size distribution of the scatterers, so they often are found empirically.

**Layered Atmosphere Model**

Practical solutions of at-sensor radiance come from models that represent the atmosphere as a set of layers with thickness \( l \), each obeying Beer’s Law:

\[
T = \left( \frac{L(\lambda, l)}{L(\lambda, 0)} \right) = e^{-\int a \rho dz} = e^{-\alpha_a dz}
\]

Since all the coefficients depend on the density of the atmospheric constituents, the type of constituent, and the optical wavelength, we divide the graded atmosphere into \( j \) homogeneous, discrete layers of thickness \( z_j \) and sum over each of \( i \) constituents:

\[
T = e^{-\sum_j \left( \sum_i (\alpha_{aij} + \beta_{Rij} + \beta_{nij} + \beta_{aij})z_j \right)}
\]
Atmospheric Radiative Transfer Modeling

Radiative transfer codes, such as MODTRAN, model atmospheric radiance and transmission for sensor system design or analysis. This image shows calculated solar radiance for a summer sky at a mid-latitude location. Notice there is more radiation on the blue side (Rayleigh scattering).

MODTRAN is a moderate-resolution code with spectral resolution of ~ 1 cm\(^{-1}\) (it can also provide band-integrated values for a user-defined sensor band), and FASCODE is its high-resolution cousin, appropriate for laser sensors.

MODTRAN and FASCODE are products of the U.S. Air Force Research Laboratory [commercially available through Ontar (www.ontar.com) as PCMODWIN and PCInWin].

MODTRAN & FASCODE Computer Models

FASCODE and PCMODWIN can both model atmospheric transmittance. FASCODE is a line-by-line radiative transfer code that uses the HiTran spectroscopic database, so it offers the highest possible spectral resolution. MODTRAN uses a band model approximation to HiTran. The examples below are for a slant path from the ground at 89° zenith angle (5-km visibility).
MODTRAN & FASCODE Computer Models (2)

FASCODE does not include scattered sunlight, so it can only be used to calculate atmospheric radiance in spectral bands dominated by thermal emission. This example is a slant path from 0 to 30 km at 10 zenith angle in a 1976 U.S. Standard Atmosphere.

MODTRAN & FASCODE Computer Models (3)

Do not use FASCODE to calculate radiance in non-thermal spectral bands…