a) The storage and loss curves for a polymer as a function of frequency are typically plotted on a log-log plot. There are several reasons for this.
   - **Explain** how the range of values typically observed for G' leads to the use of a log scale.
   - **Explain** how the functional form of J" for a single relaxation time Debye transition indicates the use of a log frequency axis.

b) Show that the Arrhenius temperature dependence indicates the use of a log frequency scale by
   - **Writing** the Arrhenius function for the relaxation time
   - **converting** this to a log function
   - **discussing** the relationship between the relaxation time and frequency for a simple Debye transition at two temperatures, $T_1$ and $T_2$.

c) **Sketch** a typical time dependent modulus versus time and time dependent compliance versus time curve.
   - **Give a function** that describes time dependent compliance as a function of time at very long times in the terminal region.
   - **Give a function** that describes the time dependent modulus as a function of time at very long times.
   - **If you used** a log axis for the time scale in your sketch explain why based on the functions you gave and the typical range of time involved for a full curve.

d) **Show** how the Arrhenius shift factor $a_T$ can be obtained form the Arrhenius temperature dependence function.
   - **Compare** this shift factor with the WLF shift factor. Under what condition is the WLF equation the same as the Arrhenius equation.

e) The WLF function for the $\alpha$-transition contains a constant, $C_2$, that is a temperature 30 to 70 degrees below $T_g$.
   - **What** is special about the glass transition that requires this temperature when compared to other lower temperature transitions such as $\beta$- or $\gamma$-transitions?
   - **Is it possible** to observe the glass transition below $C_2$ at very long times or very low frequencies?
   - **Is it possible** to observe $\beta$- or $\gamma$-transitions at very low temperatures well below the normal transition temperature for long times or very low frequencies?
a) Typically $G'$ spans 9 orders of magnitude. If a log scale were not used the low values would be swamped in a plot and only the high modulus end could be observed.

For a single relaxation time Debye transition $J'' = \Delta J \omega \tau/(1 + \omega^2 \tau^2) = \Delta J/(10^{\log(\omega \tau)} + 10^{\log(\omega \tau)})$ so the loss peak will be symmetric in $\log(\omega \tau)$.

b) $\tau = \tau_0 \exp(-E_a/kT)$ taking a log of both sides $\log \tau = \log \tau_0 - E_a/kT$

In the equations for a simple Debye relaxation $\tau$ and $\omega$ always appear as a pair so for observation at two temperatures, $T_1$ and $T_2$, the measurement at $T_2$ can be converted to an equivalent frequency at $T_1$ by using $\log(\tau_1 \omega) = \log(\tau_0 \omega) - E_a/kT_1$ that gives the Arrhenius shift factor equation, on a low frequency scale, $\ln a_T = (E_a/k)(T_2^{-1} - T_1^{-1})$.

c) 

![Graph of log Modulus versus log time and log Compliance versus log time](image)

At long times $J(t) = t/\eta$, and $G(t) = J_0/\eta_0^2 t^2$. The log time scale is used because the time range is 20 orders and the functions at long times in the terminal zone are power-law functions that are naturally presented on a log-log plot to make the power-law regions appear linear.

d) For two temperatures, $T_1$ and $T_2$, we can write $\log(\tau_1 \omega) = \log(\tau_0 \omega) - E_a/kT_1$ and $\log(\tau_2 \omega) = \log(\tau_0 \omega) - E_a/kT_2$. Subtracting the second from the first we obtain, $\log(a_T) = \log(\tau_2 \omega) - \log(\tau_1 \omega) = (-E_a/k)(1/T_2 - 1/T_1)$.

The WLF function is $\log(a_T) = -C_1 (T - T_0)/(C_2 + T - T_0)$, the difference between the two equations involves the second parameter $C_2$. The two functions are identical when $C_2 = T_0$. 

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e) The glass transition has a finite temperature limit when it is considered a true second order transition. That is, below $C_2$ the $\alpha$-transition can not occur. This is not true of simple Arrhenius transitions such as $\beta$- or $\gamma$-transitions. For these simple transitions the transitions occur for all temperatures above absolute 0. When viewed as a true second order transition it is not possible to observe the $\alpha$-transition below $C_2$. It is possible to observe the $\beta$- or $\gamma$-transitions for all temperatures above absolute 0 by looking at very long times or very low frequencies.