030207 Quiz 3 Polymer Processing

1) The Reynolds Equation,
\[ v_1(y) = \frac{1}{2\eta} \frac{\partial P}{\partial y} y(v - H) + V_0 \frac{y}{H} \]
is obtained from the Navier-Stokes Equation applied to simple shear flow between two parallel plates. Only the x-component of the Navier-Stokes Equation was used to obtain \( v_1(y) \) in class,
\[ \rho \left( \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = -\frac{\partial P}{\partial x} + \eta \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) + \rho g, \quad (x\text{-part of NSE}) \]
a) Many of the terms in the Navier-Stokes equation can be set to 0 by simple assumptions. List (with reference to the affected term) the assumptions involved in obtaining the simplified x-component of the Navier-Stokes equation given below.
\[ \frac{\partial P}{\partial x} = \frac{\partial}{\partial y} \left( \frac{\partial \tau_{xy}}{\partial y} \right) = \eta \left( \frac{\partial^2 v_x}{\partial y^2} \right) \]
b) Through integration and application of limits a simple description of the x-velocity is obtained,
\[ v_1(y) = \frac{1}{2\eta} \frac{\partial P}{\partial y} y(v - H) + V_0 \frac{y}{H} \]
Sketch the x-velocity profile as a function of y for the two terms in this expression as well as for the entire Reynolds Equation under an applied pressure gradient and a non-zero plate velocity.

2) The Weissenberg number is a dimensionless number that has been used to estimate the number of entanglements in a polymer melt, \( We = \psi_1/\tau_{12} \), where \( \psi_1 \) is the first normal stress difference, \( \psi_1 = (\tau_{11} - \tau_{22}) \) and \( \tau_{12} \) is the shear stress.
   a) Use simple constitutive equations for the first normal stress difference and the viscosity to obtain an expression for \( We \) in terms of the rate of strain and constitutive constants.
   b) Use this expression and the original definition of the Weissenberg number to explain why "We" might be used as a measure of the number of entanglements.

3) Polymer fluids display steady state rheological features as well as transient rheological features. For example, under steady shear flow the fluid displays a viscosity and a first normal stress difference which are both steady state features. Additionally, if the shear stress is removed the fluid will recoil with a recoverable shear compliance, \( J_e \).
   a) Sketch on a log-log plot \( \eta, \Psi_1 \) and \( \gamma_e \) versus the rate of strain.
   b) Write an expression relating the recoverable shear compliance at low strain rate, \( J_e \), to the recoverable shear strain and to the shear stress.
   c) Give an expression that relates \( \eta_0, \Psi_{1,0} \) and \( J_{e,0} \).
d) The expression you wrote for c) means that a polymer fluid displays only two rheological signatures, drag or viscous forces and 3D connectivity or elasticity that lead to normal forces. If the fluid were a 3D transient network (entanglements equally form and dissolve with time) explain in your own words why the recoverable strain would be related to $\eta_0$ and $\Psi_{1,0}$.

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1) a.) Steady State- First term is 0
Flow only in x direction 3'rd and 4'th term 0
No elongational flow, second term is 0, on right hand side 2'nd term is 0
Infinite plates (no change in x velocity in z direction) 4'th term on RHS is 0
No gravity effects, last term 0.

The equation in terms of shear stress depends on Newton's constitutive equation.

b.)

2) $We = \Psi_\gamma / \eta (d\gamma/dt)$. This could be used to estimate entanglements since the presence of entanglements and orientation of chains causes normal stress differences to occur. $We = 0$ means no entanglements, $We = high$ means many entanglements. The equation indicates that entanglements are associated with high rates of strain and a high ratio between the first normal stress coefficient and the shear viscosity.

The crucial feature is that normal stress results only when there is an entanglement network to transfer shear strain to normal forces. Without entanglements there is no mechanism to develop normal forces. The ratio makes sense since the viscosity reflects in some sense the fraction of shear strain that is transferred to shear stress.
3) a) 

b) \( J_e(t) = \frac{\gamma_e(t)}{\tau} \)

c) \( J_{e,0} = \frac{\Psi_{1,0}}{2\eta_0} \)

d) 3D connectivity results in normal forces from shear strain as discussed above. Additionally, the presence of a transient network makes the fluid act as if it is an elastic solid, at least at short times before the memory of fading entanglements disappears. Then there are only two mechanistic concepts that are needed to explain viscosity, normal stresses and recoverable strain, 1) viscous Eyring behavior and 2) a transient network that a) stores elastic energy and b) transfers shear strain to normal forces.