1) The figure above shows a bubble rising in a fluid.
   a) From this figure can you say if a Newtonian constitutive equation is appropriate for viscosity? Why?
   b) Explain the behavior observed in the first figure and give a constitutive equation that might be used to predict the extent of this behavior.
   c) Explain the behavior observed in the second figure and give a constitutive equation that might be used to predict the extent of this behavior.
   d) Would a single capillary viscometry measurement, melt index, be useful in determining if this is a Newtonian fluid? Explain.

2) In the fiber spinning lab we extruded polymer and then drew the extrudate using a spool. Explain how the viscometric equations for each of these viscometers could be used to model the processing flows involved in fiber spinning.
   a) Couette viscometer.
   b) Capillary viscometer
   c) Cone and plate viscometer
   d) Describe a viscometer that could be used to model the drawing of the fiber after extrusion. What type of flow is this?

3) The cone and plate viscometer has distinct advantages for viscometric characterization of non-Newtonian fluids.
   a) What is the main advantage of a cone and plate viscometer over a Couette or capillary viscometer?
   b) For a cone and plate viscometer the cone rotates at angular velocity, $\Omega$, and has a cone angle of $\theta_0 = 1^\circ$. Calculate the shear rate as a function of radial position $r$ for this viscometer.
   c) Calculate the shear stress for this viscometer if a torque $T$ is measured on the bottom plate and the fluid covers an area of radius $R$.
   d) Give an expression for the viscosity for this viscometer as measured at the bottom plate.
   e) Explain how the first normal stress difference could be calculated for the cone and plate viscometer.
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1) a) There is no way to tell if the fluid is Newtonian or non-Newtonian since there is no indication of the dependence of viscosity on rate of strain.
   b) The bubble is non-spherical due to the presence of a first normal stress difference. The fluid is sheared at the sides of the bubble and a normal force develops due to this shear that results in a normal force which pinches the bubble at the bottom. The first normal stress difference can be expressed using the following constitutive equation,

   \[ \psi_1 = \Psi_1 \dot{\gamma}^2 \]

c) The second figure shows the presence of a second normal stress difference since the bubble is asymmetric. The second normal stress difference is usually described using a similar function,

   \[ \psi_2 = \Psi_2 \dot{\gamma}^2 \]

d) A capillary viscometry measurement would be of no use in determining if the fluid is non-Newtonian since the rate of strain varies across the capillary in this measurement.

2) a) The flow between the flights of the extruder involves annular shear flow that can be modeled in the Couette viscometer.
   b) The flow in a capillary viscometer involves Pousiulle flow in a tube which mimics the flow in the extruder die.
   c) The cone and plate viscometer does not model any of the processing flows in fiber spinning.
   d) Drawing of the fiber after spinning can be modeled in a uniaxial extensional viscometer. This could be studied at the convergence point of a 6 pipe system for instance. Four tubes converge their flows at a point and two axial pipes drain the flow. The flow rate in the exiting pipes is twice the flow rate in the entering pipes. This flow is uniaxial extensional flow.

3) a) The main advantage is that the shear rate is constant for all fluid points in the viscometer. This means that the shear rate dependencies of the viscosity can be directly measured in this viscometer.
   b) The shear rate is not a function of \( r \) so calculation of the shear rate at any point in the viscometer will suffice. For instance, at a distance \( r \) from the center the velocity is \( \Omega r \) at the cone and 0 at the plate. The gap size at this point is \( r \sin\theta_0 \approx r\theta_0 \) so the rate of strain is \( \dot{\gamma} = \Omega \theta_0 \).
   c) If the shear rate is the same across the gap then the shear stress must be the same also. The differential F/A is

   \[ \tau = \frac{3T}{2\pi R^3} \]

   d) The viscosity is the ratio of b) and c),

   \[ \eta = \frac{3T\theta_0}{2\pi R^2 \Omega} \].
e) The first normal stress difference can be calculated from the force pressing down on the bottom plate. \( \psi_1 = \frac{2F}{\pi R^2} \).