At low-q the Debye equation for polymer coils can be approximated by Guinier's Law,

\[ S(q) = G e^{-\left(q^2R_g^2/3\right)} \approx G \left(1 - \frac{q^2R_g^2}{3}\right) \]

This can also be expressed, \(1/S(q) = G^{-1}(1 + q^2R_g^2/3)\)

Using \(G^{-1} = (1/kT) (\delta\Pi/\deltac)_T = 1/N + 2A_c\), we arrive at the Zimm Equation,

\[ S(q) \text{ is proportional to the scattered intensity} \]

The Zimm Equation (and plot) have a close relationship with the RPA equation for polymer blends.

\[ \text{a) Give the RPA equation for a binary polymer blend with thermal interactions.} \]

\[ \text{Write the Zimm equation in a similar format.} \]

\[ \text{How do the two equations compare?} \]

\[ \text{b) The Zimm plot is a series of plots of } 1/S(q) \text{ versus } q^2 + kc \text{ where } k \text{ is an arbitrary constant.} \]

\[ \text{What is the logic behind such a plot?} \]

\[ \text{c) In the RPA equation if the Zernike function is used rather than the Debye function can a plot similar to the Zimm plot be used?} \]

\[ \text{d) In the derivation of the RPA equation an external field } \Psi_k \text{ was applied to polymer component A in a binary blend. The definition of } \Psi_k \text{ is somewhat vague.} \]

\[ \text{Why is this not important to the outcome of the derivation?} \]

\[ \text{e) Resistors in series add, } R = R_A + R_B \text{ while resistors in parallel follow } 1/R = 1/R_A + 1/R_B. \]

\[ \text{Explain an analogy that is possible between the RPA equation and resistors.} \]

\[ \text{Which is more important, the strongest or the weakest scatter for a polymer blend?} \]

\[ \text{How would this differ for an immiscible mixture composed of A and B phases?} \]

\[ \text{What is the basis of this behavior in the RPA equation (from the derivation in class)?} \]
a) \[ \frac{1}{S(q)} = \frac{1}{\phi_A N_A S_D(R_A^2 q^2)} + \frac{1}{\phi_B N_B S_D(R_B^2 q^2)} - 2\chi_{AB} \]

\[ \frac{1}{S(q)} \approx G^{-1} \frac{1}{S_D(R^2 q^2)} = \frac{1}{N S_D(R^2 q^2)} + 2A_2c \]

The two equations are of the same form. The Zimm equation is a limit of the RPA equation for low molecular weight in component B.

b) \[ \frac{1}{S(q)} \] is linear in \( q^2 \) but also depends on concentration. In order to extrapolate to low concentration and low q the Zimm plot is made.

c) If \[ S(q) \approx \frac{1}{1 + q^2 \xi^2} \] then \[ \frac{1}{S(q)} \] is linear in \( q^2 \) so a plot of \( \frac{1}{S(q)} \) versus \( q^2 \) is appropriate.

d) The field is not important because it cancels out of the derivation, i.e. it is not part of the RPA equation in the end.

e) The RPA is similar to parallel resistors in that it is the inverse of the intensities that sum. This will lead to a stronger contribution by the weaker component just as most of the current flows through the weakest resistor in parallel. The series case is similar to the case where scattering from the two components is independent. It is the coupling of concentration fluctuations that leads to the parallel function in the RPA.