a) The Finger tensor for uniaxial extension is given by:

\[
B_{\alpha\beta} = \begin{pmatrix}
\sqrt{\lambda} & 0 & 0 \\
0 & \sqrt{\lambda} & 0 \\
0 & 0 & \lambda^2
\end{pmatrix}
\]

Give the expression for the stress tensor in a bulk rubber and From this give an expression for the tensile stress \( \sigma_{zz} \) in terms of \( \lambda \). How does this relate to the Mooney-Rivlin Plot?

b) For simple shear extension the Finger tensor is:

\[
B_{\alpha\beta} = \begin{pmatrix}
(\gamma^2 + 1) & \gamma & 0 \\
\gamma & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

Write expressions for the shear stress, \( \sigma_{xy} \), and the first, \( \sigma_{xx} - \sigma_{zz} \), and the second, \( \sigma_{xx} - \sigma_{yy} \), normal stress differences. Are these results consistent with Hookean behavior?

c) Shear flow can be described using the equation of state for a rubber (expression for stress from question "a") with a constant rate of shear strain (question "b") and applying the Boltzmann superposition position principle. The Boltzmann principle states that the present state of stress (or other property) is a result of the accumulated strains times a time dependent modulus (memory function).

Do you expect this approach to yield a first normal stress difference, \( \sigma_{xx} - \sigma_{zz} \)? Why?
Do you expect this approach to yield a second normal stress difference, \( \sigma_{xx} - \sigma_{yy} \)? Why?

d) The Stress-Optical Law states that \( \Delta n = C_{opt} (\sigma_{zz} - \sigma_{xx}) \) Doi/Edwards p. 222

i. Does \( C_{opt} \) depend on temperature?
ii. Does \( C_{opt} \) depend on molecular weight?
iii. Does \( C_{opt} \) depend on branching or cross link density?
iv. Does \( C_{opt} \) depend on polymer concentration in a concentrated solution?
v. Does \( C_{opt} \) depend on the relationship between stress and shear rate in flow?

e) The Stress-Optical Law indicates that stress and birefringence (\( \Delta n \)) have the same physical origin. Doi/Edwards p. 222
What is the physical origin that unites these two properties?
a) \[ \sigma = G \cdot B - P \mathbf{I} \]

\[ \sigma_{zz} = \nu_c kT \lambda^2 - P \]

\[ \sigma_{xx} = \sigma_{yy} = \nu_c kT / \lambda - P \]

for \( \sigma_{xx} = \sigma_{yy} = 0, P = \nu_c kT / \lambda, \) and

\[ \sigma_{zz} = \nu_c kT (\lambda^2 - 1 / \lambda) \]

The latter equation is the basis of the Mooney-Rivlin Plot (Strobl p. 323) of reduced tensile stress, \( \sigma / (\lambda - 1 / \lambda^2) \), versus the inverse of strain, \( 1 / \lambda \).

b) \[ \sigma_{xy} = (n_c / V) kT \gamma \]

\[ \sigma_{xx} - \sigma_{zz} = (n_c / V) kT \gamma^2 \]

\[ \sigma_{yy} - \sigma_{zz} = 0 \]

The expression for the shear stress is Hook's Law for shear if \( (n_c / V) kT \) is constant in strain. The first normal stress difference is not consistent with Hook's Law since Hookean behavior does not accommodate a normal stress difference. The second normal stress difference is consistent with Hook's law.

c) A first normal stress difference is expected since question b results in a first normal stress difference. A second normal stress difference is not expected since question b does not predict a second normal stress difference.

d) \( C_{opt} \) is linear in temperature

\( C_{opt} \) does not depend on molecular weight

\( C_{opt} \) does not depend on cross linking or branching

\( C_{opt} \) does depend on polymer concentration

\( C_{opt} \) does not depend on the relationship between stress and strain rate.

e) "Orientational ordering in the polymer segments" Doi/Edwards p. 222 The theory of polymer dynamics.