1) The time auto-correlation function, \( g^1(q, \tau) \) is used to obtain the diffusion coefficient from a noise pattern created by the change in scattered intensity with time from a colloidal suspension.

a) Sketch a plot the intensity as a function of time for large particles, medium sized particles and small colloidal particles. Are these curves different? Why?

b) What is the time correlation function \( g^2(q, \tau) \), show how it is determined from an intensity versus time plot.

c) What is the value of \( g^2(q, \tau) \) when \( \tau = 0 \) and when \( \tau = \infty \)?

d) Write an expression for \( g^1(q, \tau) \) as a function of \( g^2(q, \tau) \) using your answer to “c” to define the parameters.

e) How can the hydrodynamic radius be obtained from \( g^1(q, \tau) \)?

2) The following scattering function has been proposed by Benoit [H. Benoit, J. Polym. Sci., 1953, XI, 507] for scattering from Gaussian star polymers (\( f \) is the number of arms, \( b \) is the segment length, \( n \) is the number of segments per arm). (Star polymers are polymers with arms emanating from a center point.)

\[
S_{Star}(q, b, n, f) = \frac{P_{11}}{f} + \frac{f - 1}{f} P_{12}
\]

\[
P_{11}(q, b, n, f) = \frac{2}{(xn)^2} \left( e^{-xn} - 1 + xn \right)
\]

\[
P_{12}(q, b, n, f) = \frac{(1 - e^{-xn})^2}{(xn)^2}
\]

\[
x = \frac{(qb)^2}{6}
\]

a) Show that the first term \( (P_{11}) \) displays Gaussian scaling at high-q.

b) Obtain the radius of gyration for the first term by extrapolating the first term to low-q and comparing with Guinier’s law.

c) Explain what part of the star structure you think the first term describes.

d) Show that the second term is non-fractal in nature (fractal structures display a power-law decay between -1 and -3).

e) Do you think that this function could describe a star polymer with Gaussian arms? Explain your answer.
1) a) 

\[ \frac{g^2(q, \tau)}{\langle I(t) \rangle^2} = \frac{\langle I(t) I(t+\tau) \rangle}{\langle I(t) \rangle^2} \]

Compare two intensities separated by \( \tau \) multiply these two intensities and take an average over all stochastic times, \( t \). This average at \( \tau \) is normalized by the value at \( \tau = 0 \).

b) 

\[ g^2(q, \tau) = 1 \quad \text{at} \quad \tau = 0 \quad \frac{\langle I(t) \rangle^2}{\langle I(t) \rangle} \]

So \( g^2(q, \tau) = \frac{\langle I(t) \rangle^2}{\langle I(t) \rangle^2} \)
\[ d) \quad q^2(g, \tau) = 1 + \beta (q'(g, \tau))^2 \]
\[ \beta = \left( \frac{\langle I^2(t) \rangle}{\langle I(t) \rangle^2} - 1 \right) \]

\[ c) \quad q'(g, \tau) = \exp(-q^2 0 \tau) \]

\[ D = \frac{kT}{6\pi\eta R_h} \]

2) a) at high\( q \quad e^{-x_n} \gg 0 \quad \& \quad x_n - 1 \equiv x_n \]
\[ p_{ii}(g \to 0) \sim \frac{2}{x_n} = \frac{2\langle G \rangle}{q^2 n b^2} \quad d_f = 2 = \text{Gaussian} \]

b) at low\( q \quad e^{-x_n} \gg 1 \sim x_n x_n + (x_n)^2 + (x_n)^3 + \ldots \]
\[ p_{ii}(g \to 0) \sim 2 \left( 1 - \frac{x_n}{8} + \ldots \right) \sim \exp\left( -\frac{x_n}{3} \right) \]

\text{Gumbel's Law} \quad \exp\left( \frac{-x_n}{3} \right) \quad \therefore \quad \frac{q^2}{2} = \frac{n b^2}{6} \]
c) The first term describes scattering from the arms as independent contributions to the overall scattering pattern.

\[ d) \quad \text{at high } q, \quad e^{-x_n} \rightarrow 0 \]

\[ \rho_{12} \Rightarrow \frac{1}{(x_n)^2} = \frac{36}{q^4 5n^2} \]

\[ \rho_{12} \sim q^{-4} \text{ so this is a non-trivial scattering law} \]

\[ e) \quad \text{there should be no part of the star scattering that would display a } q^{-4} \text{ scaling so this function is erroneous.} \]