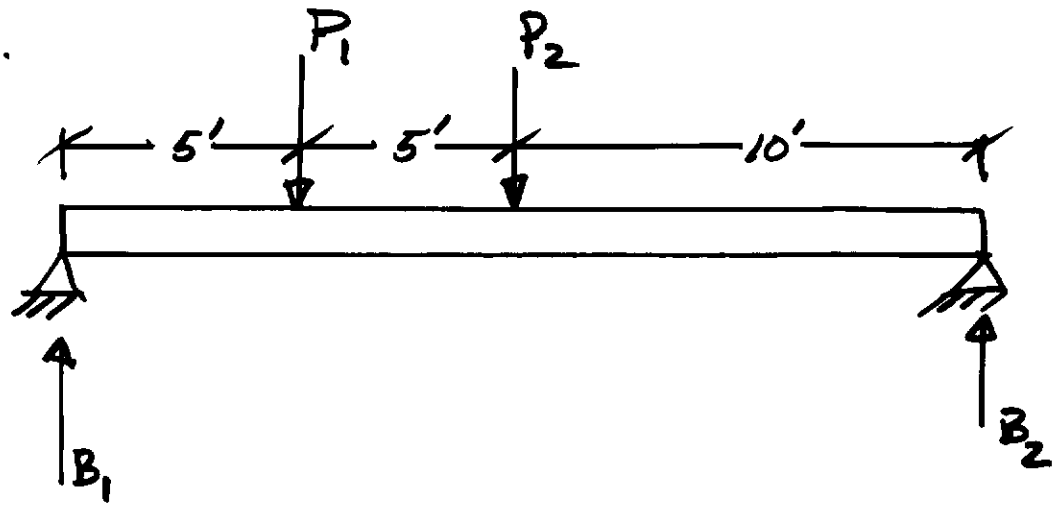
3.22.



DATA: Load Carrying Cap. of Bearing 1 = 12 kips
" " " " " 2 = 19 kips
Maximum bending moment = 65 kip-ft
Assume max bending moment occurs at one of 2 loading points.

DETERMINE: Maximum possible load P_1 & P_2 given strength of beam & supports.

Step 1: Beam Analysis

$$\Sigma F_y = 0 = B_1 + B_2 - P_1 - P_2$$
$$\Sigma M_1 = 0 = 5P_1 + 10P_2 - 20B_2$$

} A Descriptive model of structure —

Answers the question: What are equilibrium forces on structure, given loads?

Step 2: Embed descriptive model with an optimal design model

Maximize $Z = P_1 + P_2$
P_1, P_2, B_1, B_2

Subject to

$$\left.\begin{aligned} B_1 + B_2 - P_1 - P_2 &= 0 \\ -20B_2 + 5P_1 + 10P_2 &= 0 \end{aligned}\right\}$$ Structural Analysis Model (constraints)

$$\left.\begin{aligned} B_1 &\le 12 \\ B_2 &\le 19 \end{aligned}\right\}$$ Bearing Capacity

$$\left.\begin{aligned} 5B_1 &\le 65 \\ 10B_2 &\le 65 \end{aligned}\right\}$$ Max. moment Capacity.

But Max Bending Moment Constraints can also be written:

$$B_1 \le 13$$
$$B_2 \le 6.5$$

And so the model can be simplified by recognizing that one of the bearing cap. & moment capacity constraints will never be binding, & thus may be neglected. Doing so, and writing the remaining constraints in equality form:

Maximize $Z = P_1 + P_2$
$P_1, P_2, B_1, B_2, S_3, S_4$

s.t.

$$\begin{aligned} B_1 + B_2 - P_1 - P_2 &= 0 \\ -20B_2 + 5P_1 + 10P_2 &= 0 \\ B_1 + S_3 &= 12 \\ B_2 + S_4 &= 6.5 \end{aligned}$$

$$B_1, B_2, P_1, P_2, S_3, S_4 \ge 0$$

Note that slack/surplus variables not needed for constraints that are originally in equality form.