$\boxed{3.2}$ Min $Z = 4x_1 - 5x_2$

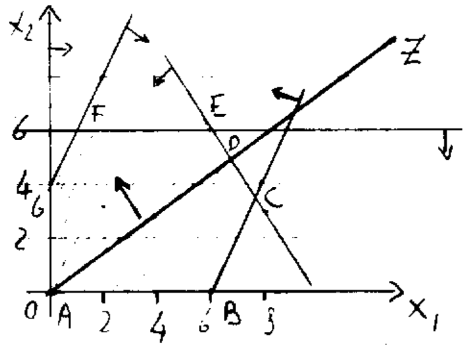
s.t.: $-6x_1 + 3x_2 \leq 12$

$4x_1 - 2x_2 \leq 24$

$3x_1 + 2x_2 \leq 30$

$x_2 \leq 6$

$x_1, x_2 \geq 0$



- Basis Representation for $B^A = \{S_1, S_2, S_3, S_4\}$

$S_1 = 12 + 6x_1 - 3x_2$

$S_2 = 24 - 4x_1 + 2x_2$

$S_3 = 30 - 3x_1 - 2x_2$

$S_4 = 6 - x_2$

$Z = 0 + 4x_1 - 5x_2$

- x_2 enter the basis (we are minimizing)

$S_1 \geq 0 \iff 12 - 3x_2 \geq 0, \; x_2 \leq 4$ $\quad$ S_1 leaves basis

$S_2 \geq 0 \iff 24 + 2x_2 \geq 0 \; ; \; \forall x_2 \in [0, \infty[$

$S_3 \geq 0 \iff 30 - 2x_2 \geq 0 \; ; \; x_2 \leq 15$

$S_4 \geq 0 \iff 6 - x_2 \geq 0 \; ; \; x_2 \leq 6$

$\Rightarrow \quad x_2 = (12 + 6x_1 - S_1)/3$

- Basis Representation for $B^6 = \{x_2, S_2, S_3, S_4\}$

$$
\begin{array}{l||l}
x_2 = 4 + 2x_1 - \frac{1}{3}S_1 & x_2 = 4 + 2x_1 - \frac{1}{3}S_1 \\
S_2 = 24 - 4x_1 + 2\left(4 + 2x_1 - \frac{1}{3}S_1\right) & S_2 = 32 - \frac{2}{3}S_1 \\
S_3 = 30 - 3x_1 - 2\left(4 + 2x_1 - \frac{1}{3}S_1\right) & S_3 = 22 - 7x_1 + \frac{2}{3}S_1 \\
S_4 = 6 - \left(4 + 2x_1 - \frac{1}{3}S_1\right) & S_4 = 2 - 2x_1 + \frac{1}{3}S_1 \\
\hline
Z = 4x_1 - 5\left(4 + 2x_1 - \frac{1}{3}S_1\right) & Z = -20 - 6x_1 + \frac{5}{3}S_1
\end{array}
$$

- x_1 enter the basis

$x_2 \geq 0 \iff 4 + 2x_1 \geq 0 \; ; \; \forall x_1 \in [0, \infty[$

$S_2 \geq 0 \iff \forall x_1 \in [0, 8[$

$S_3 \geq 0 \iff 22 - 7x_1 \geq 0 \; ; \; x_1 \leq \frac{22}{7}$

$S_4 \geq 0 \iff 2 - 2x_1 \geq 0 \quad x_1 \leq 1$ $\quad S_4$ leaves basis

$$\Rightarrow \quad x_1 = \left(2 + \frac{1}{3}S_1 - S_4\right)/2$$

- Basis Representation for $B^F = \{x_2, S_2, S_3, x_1\}$

$$
\begin{array}{l||l}
x_2 = 4 + \left(2 + \frac{1}{3}S_1 - S_4\right) - \frac{1}{3}S_1 & x_2 = 6 - S_4 \\
S_2 = 32 - \frac{2}{3}S_1 & S_2 = 32 - \frac{2}{3}S_1 \\
S_3 = 22 - 7\left(2 + \frac{1}{3}S_1 - S_4\right)/2 + \frac{2}{3}S_1 & S_3 = 15 - \frac{1}{2}S_1 + \frac{7}{2}S_4 \\
x_1 = 1 + \frac{1}{6}S_1 - \frac{1}{2}S_4 & x_1 = 1 + \frac{1}{6}S_1 - \frac{1}{2}S_4 \\
\hline
Z = -20 - 6\left(1 + \frac{1}{6}S_1 - \frac{1}{2}S_4\right) + \frac{5}{3}S_1 & Z = -26 + \frac{2}{3}S_1 + 3S_4
\end{array}
$$

$\Rightarrow$ STOP

$$F \begin{cases} x_1 = 1 \\ x_2 = 6 \end{cases} \qquad Z = -26$$

Unique Optimal Solution

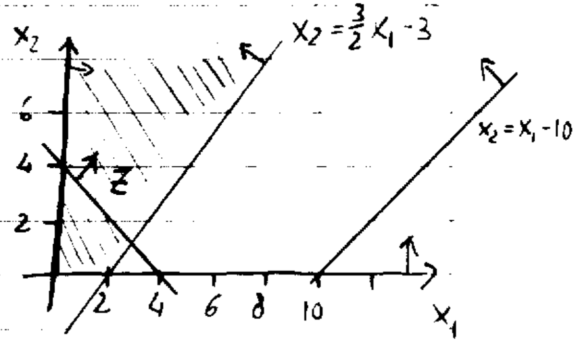
$\boxed{3.5}$ $\text{Max } Z = x_1 + x_2$

s.t. $x_1 + x_2 \geq -4 \;;\; -x_1 - x_2 \leq 4$

$3x_1 - 2x_2 \leq 6$

$x_1 - x_2 \leq 10$

$x_1 , x_2 \geq 0$



* Basis Representation $B^A = \{S_1, S_2, S_3\}$, $A = (0,0)$

$S_1 = +4 + x_1 + x_2$

$S_2 = 6 - 3x_1 + 2x_2$

$S_3 = 10 - x_1 + x_2$

$Z = 0 + x_1 + x_2$

- x_2 enter Basis

$S_1 \geq 0 \iff 4 + x_2 \geq 0 \quad \forall\, x_2 \in [0, \infty[$

$S_2 \geq 0 \iff 6 + 2x_2 \geq 0 \quad \forall\, x_2 \in [0, \infty[$

$S_3 \geq 0 \iff 10 + x_2 \geq 0 \quad \forall\, x_2 \in [0, \infty[$

$\Rightarrow$ STOP   This optimization problem is unbounded

$\boxed{3.16}$ $X_1 = \#/100$ Tees
$X_2 = \#/100$ Elbows
$X_3 = \#/100$ Splicers

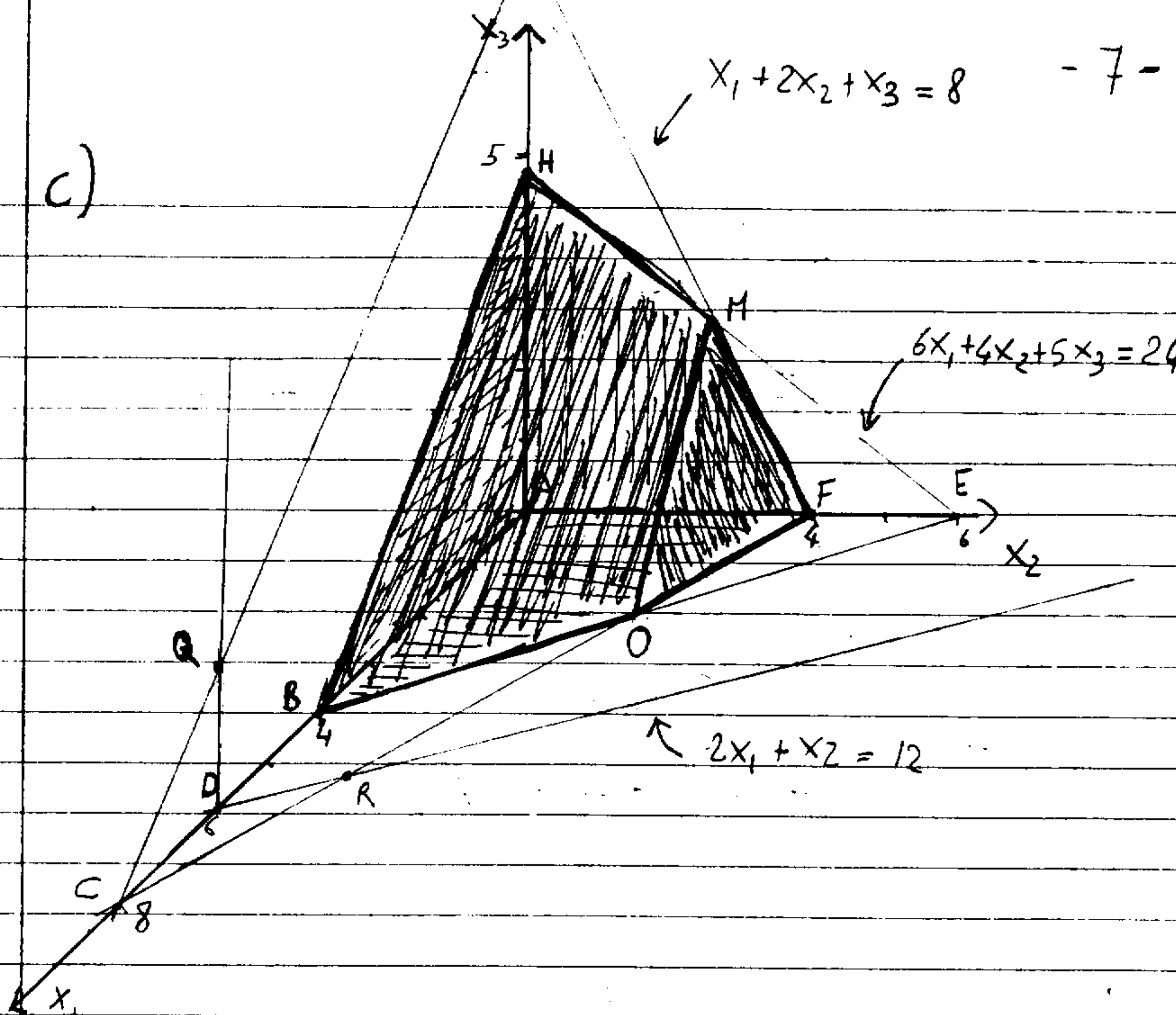
a) Maximize $Z = 700X_1 + 550X_2 + 480X_3$

s.t. $6X_1 + 4X_2 + 5X_3 \leq 24$
$X_1 + 2X_2 + X_3 \leq 8$
$2X_1 + X_2 \leq 12$
$X_1, X_2, X_3 \geq 0$

b) $6X_1 + 4X_2 + 5X_3 + S_1 = 24$
$X_1 + 2X_2 + X_3 + S_2 = 8$
$2X_1 + X_2 + S_3 = 12$

	X_1	X_2	X_3	S_1	S_2	S_3	Z		
A	0	0	0	24	8	12	0		
B	4	0	0	0	4	4	2800		
C	8	0	0	−24	0	−4	—	*	(* Not feasible)
D	6	0	0	−12	2	0	—	*	
E	0	6	0	0	−4	6	—	*	
F	0	4	0	8	0	8	2200		
G	0	12	0	−24	−16	0	—	*	
H	0	0	$\frac{24}{5}$	0	$\frac{16}{5}$	12	2304		
i	0	0	8	−16	0	12	—	*	
L	$+\frac{56}{13}$	$+\frac{44}{13}$	$-\frac{40}{13}$	0	0	0	—	*	
M	0	+8/3	8/3	0	0	28/3	2747		
N	−16	0	24	0	0	44	—	*	
⇒ O	2	3	0	0	0	5	3050	⇐	
P	0	12	−16	56	0	0	—	*	
Q	6	0	2	−22	0	0	—	*	
R	16/3	4/3	0	−40/3	0	0	—	*	
S	0	12	−24/5	0	−56/5	0	—	*	
	6	0	−12/5	0	22/5	0	—	*	
T	12	−12	0	0	−20	0	—	*	

c)



d) Basis Representation for $B^A = \{S_1, S_2, S_3\}$

$$S_1 = 24 - 6X_1 - 4X_2 - 5X_3$$
$$S_2 = 8 - X_1 - 2X_2 - X_3$$
$$S_3 = 12 - 2X_1 - X_2$$

$$Z = 700X_1 + 550X_2 + 480X_3$$

- X_1 enter basis

$S_1 \geq 0 \iff 24 - 6X_1 \geq 0 \;;\; X_1 \leq 4$ $\quad$ S_1 leaves basis

$S_2 \geq 0 \iff 8 - X_1 \geq 0 \;;\; X_1 \leq 8$

$S_3 \geq 0 \iff 12 - 2X_1 \geq 0 \;;\; X_1 \leq 6$

$$\Rightarrow X_1 = 4 - \frac{2}{3}X_2 - \frac{5}{6}X_3 - \frac{1}{6}S_1$$

- Basis Representation for $B^{\beta} = \{x_1, S_2, S_3\}$

$$x_1 = 4 - \frac{2}{3}x_2 - \frac{5}{6}x_3 - \frac{1}{6}S_1$$
$$S_2 = 8 - \left(4 - \frac{2}{3}x_2 - \frac{5}{6}x_3 - \frac{1}{6}S_1\right) - 2x_2 - x_3$$
$$S_3 = 12 - 2\left(4 - \frac{2}{3}x_2 - \frac{5}{6}x_3 - \frac{1}{6}S_1\right) - x_2$$

$$Z = 700\left(4 - \frac{2}{3}x_2 - \frac{5}{6}x_3 - \frac{1}{6}S_1\right) + 550x_2 + 480x_3$$

$$x_1 = 4 - \frac{2}{3}x_2 - \frac{5}{6}x_3 - \frac{1}{6}S_1$$
$$S_2 = 4 - \frac{4}{3}x_2 - \frac{1}{6}x_3 + \frac{1}{6}S_1$$
$$S_3 = 4 + \frac{1}{3}x_2 + \frac{5}{3}x_3 + \frac{1}{3}S_1$$

$$Z = 2800 + \frac{250}{3}x_2 - \frac{310}{3}x_3 - \frac{350}{3}S_1$$

- x_2 enter basis

$x_1 \geq 0 \iff 4 - \frac{2}{3}x_2 \geq 0, \; x_2 \leq 6$

$S_2 \geq 0 \iff 4 - \frac{4}{3}x_2 \geq 0, \; x_2 \leq 3$ $\quad$ S_2 leaves basis

$S_3 \geq 0 \iff 4 + \frac{1}{3}x_2 \geq 0, \; \forall x_2 \in [0, \infty[$

$$\Rightarrow x_2 = 3 - \frac{1}{8}x_3 + \frac{1}{8}S_1 - \frac{3}{4}S_2$$

- Basis Representation for $B^0 = \{x_1, x_2, S_3\}$

$$x_1 = 4 - \frac{2}{3}\left(3 - \frac{1}{8}x_3 + \frac{1}{8}S_1 - \frac{3}{4}S_2\right) - \frac{5}{6}x_3 - \frac{1}{6}S_1$$
$$x_2 = 3 - \frac{1}{8}x_3 + \frac{1}{8}S_1 - \frac{3}{4}S_2$$
$$S_3 = 4 + \frac{1}{3}\left(3 - \frac{1}{8}x_3 + \frac{1}{8}S_1 - \frac{3}{4}S_2\right) + \frac{5}{3}x_3 + \frac{1}{3}S_1$$

$$Z = 2800 + \frac{250}{3}\left(3 - \frac{1}{8}x_3 + \frac{1}{8}S_1 - \frac{3}{4}S_2\right) - \frac{310}{3}x_3 - \frac{350}{3}S_1$$

$$= 3050 - \frac{455}{4}x_3 - \frac{425}{4}S_1 - \frac{125}{2}S_2$$

$\Rightarrow$ STOP $\quad \begin{cases} x_1 = 2 \\ x_2 = 3 \\ x_3 = 0 \end{cases} \quad Z = 3050$ $\quad$ Unique Optimal Solution