Solution HW. #4

$\boxed{3.8}$ Minimize $Z = 2x_1 + x_2$

s.t.: $4x_1 - 12x_2 \le -6$, $2x_1 - 6x_2 \le -3$

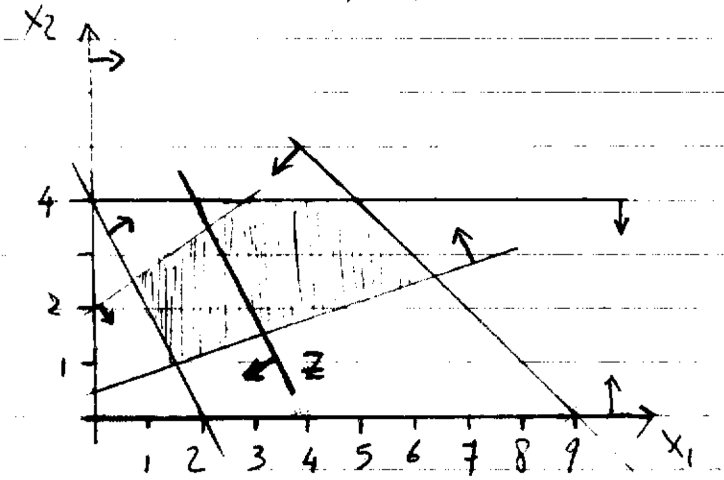
$-4x_1 + 6x_2 \le 12$, $-2x_1 + 3x_2 \le 6$

$4x_1 + 2x_2 \ge 8$, $-2x_1 - x_2 \le -4$

$x_1 + x_2 \le 9$, $x_1 + x_2 \le 9$

$4x_2 \le 16$, $x_2 \le 4$

$x_1, x_2 \ge 0$, $x_1, x_2 \ge 0$



$S_1 = -3 - 2x_1 + 6x_2$ *

$S_2 = 6 + 2x_1 - 3x_2$

$S_3 = -4 + 2x_1 + x_2$ *

$S_4 = 9 - x_1 - x_2$

$S_5 = 4 - x_2$

* Origin not a feasible point

- Auxiliary Problem: Minimize A

s.t.: $2x_1 - 6x_2 - A \le -3$

$-2x_1 + 3x_2 - A \le 6$

$-2x_1 - x_2 - A \le -4$

$x_1 + x_2 - A \le 9$

$x_2 - A \le 4$

$x_1, x_2, A \ge 0$

- Basis Representation for $B^0 = \{S_1, S_2, S_3, S_4, S_5\}$

$S_1 = -3 - 2x_1 + 6x_2 + A$

$S_2 = 6 + 2x_1 - 3x_2 + A$

$S_3 = -4 + 2x_1 + x_2 + A \rightarrow A = 4 - 2x_1 - x_2 + S_3$

$S_4 = 9 - x_1 - x_2 + A$ $\quad$ S_3 leaves basis

$S_5 = 4 - x_2 + A$

$Z = A$

Basis Representation for $B = \{S_1, S_2, A, S_4, S_5\}$

$$\begin{aligned}
S_1 &= -3 - 2x_1 + 6x_2 + (4 - 2x_1 - x_2 + S_3) \\
&= 1 - 4x_1 + 5x_2 + S_3 \\
S_2 &= 6 + 2x_1 - 3x_2 + (4 - 2x_1 - x_2 + S_3) \\
&= 10 - 4x_2 + S_3 \\
S_4 &= 9 - x_1 - x_2 + (4 - 2x_1 - x_2 + S_3) \\
&= 13 - 3x_1 - 2x_2 + S_3 \\
S_5 &= 4 - x_2 + (4 - 2x_1 - x_2 + S_3) \\
&= 8 - 2x_1 - 2x_2 + S_3 \\
A &= 4 - 2x_1 - x_2 + S_3 \\
\hline
Z &= 4 - 2x_1 - x_2 + S_3
\end{aligned}$$

x_1

$x_1 \leq 1/4$ ✓

$x_1 \leq \infty$

$x_1 \leq 13/3$

$x_1 \leq 4$

$x_1 \leq 2$

x_1 enters, S_1 leaves.

$B = \{x_1, S_2, A, S_4, S_5\}$

$$\begin{aligned}
x_1 &= 1/4 + \tfrac{5}{4}x_2 - 1/4\,S_1 + 1/4\,S_3 \\
S_2 &= 10 - 4x_2 + S_3 \\
A &= \cancel{4} - 2(1/4 + 5/4\,\cancel{x_2} - 1/4\,\cancel{S_1} + 1/4\,\cancel{S_3}) - \cancel{x_2} + \cancel{S_3} \\
&= 7/2 - 7/2\,x_2 + S_1/2 + S_3/2 \\
S_4 &= 13 - 3(1/4 + 5/4\,x_2 - S_1/4 + S_3/4) - 2x_2 + S_3 \\
&= 49/4 - 23/4\,x_2 + 3/4\,S_1 + S_3/4 \\
S_5 &= \cancel{8} - 2(\cancel{1/4} + 5/4\,x_2 - \cancel{S_1}/4 + S_3/4) - 2x_2 + S_3 \\
&= 15/2 - 9/2\,x_2 + S_1/2 + S_3/2 \\
\hline
Z &= 7/2 - 7/2\,x_2 + S_1/2 + S_3/2
\end{aligned}$$

x_2

$x_2 \leq \infty$

$x_2 \leq 5/2$

$x_2 \leq 1$ ✓

$x_2 \leq 49/23$

$x_2 \leq 15/9$

x_2 enters, A leaves.

$$B = \{x_2, x_1, S_2, S_4, S_5\}$$

$$
\begin{aligned}
x_2 &= 1 - \tfrac{2}{7}A + S_1/7 + S_3/7 \\
x_1 &= 1/4 + 5/4\,(1 - 2/7\,A + S_1/7 + S_3/7) - S_1/4 + S_3/4 \\
&= 3/2 - 5/14\,A - S_1/14 + 6/14\,S_3 \\
S_2 &= 10 - 4\,(1 - 2/7\,A + S_1/7 + S_3/7) + S_3 \\
&= 6 + 8/7\,A - 4/7\,S_1 + 3/7\,S_3 \\
S_4 &= 49/4 - 23/4\,(1 - 2/7\,A + S_1/7 + S_3/7) + 3/4\,S_1 + S_3/4 \\
&= 13/2 + 23/14\,A - 1/14\,S_1 - 8/14\,S_3 \\
S_5 &= 15/2 - 9/2\,(1 - 2/7\,A + S_1/7 + S_3/7) + S_1/2 + S_3/2 \\
&= 3 + 9/7\,A - S_1/7 - S_3/7
\end{aligned}
$$

$$
\begin{aligned}
Z &= 7/2 - 7/2\,(1 - 2/7\,A + S_1/7 + S_3/7) + S_1/2 + S_3/2 \\
&= 0 + A + 0S_2 + 0S_3
\end{aligned}
$$

Optimal basis w/ A driven to zero

Phase II – optimize true objective function.

$$B = \{x_2, x_1, S_2, S_4, S_5\}$$

$$
\begin{aligned}
x_2 &= 1 + S_1/7 + S_3/7 \\
x_1 &= 3/2 - S_1/14 + 6/14\,S_3 \\
S_2 &= 6 - 4/7\,S_1 + 3/7\,S_3 \\
S_4 &= 13/2 - 1/14\,S_1 - 8/14\,S_3 \\
S_5 &= 3 - S_1/7 - S_3/7
\end{aligned}
$$

$$
\begin{aligned}
Z &= 2\,(3/2 - S_1/14 + 6/14\,S_3) + (1 + S_1/7 + S_3/7) \\
&= 4 + 0S_1 + S_3
\end{aligned}
$$

Optimal for minimizing ✓ $Z^* = 4$.

b) Alternate optimal solution would be obtained by pivoting S_1 into basis.

3.20 X_{ij} = # of tons of material i purchased from supplier j

$i = C$ (cement), S (sand), G (gravel)

$j = A, B$

$$\text{Minimize } Z = 150\,X_{CA} + 175\,X_{CB} + 10\,X_{SA} + 7\,X_{SB} + 17\,X_{GA} + 15\,X_{GB}$$

s.t. :

$X_{CA} \le 3 \qquad X_{CB} \le 6$

$X_{SA} \le 4 \qquad X_{SB} \le 5$

$X_{GA} \le 4 \qquad X_{GB} \le 6$

$X_{CA} + X_{CB} = 2$

$X_{SA} + X_{SB} = 5$

$X_{GA} + X_{GB} = 10$

$X_{CA}, X_{CB}, X_{SA}, X_{SB}, X_{GA}, X_{GB} \ge 0$

By Inspection: $X_{CA} = 2 \qquad X_{CB} = 0$

$X_{SA} = 0 \qquad X_{SB} = 5$

$X_{GA} = 4 \qquad X_{GB} = 6$

$$Z_{min} = 300 + 35 + (17 \cdot 4 + 15 \cdot 6) = 493 \text{ \$}$$