Example 3.10

Minimize $Z = 2x_1 + 3x_2 + x_3$

s.t. $2x_1 + x_2 - x_3 \geq 3$

$x_1 + x_2 + x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$

Convert to equality form:

Minimize $Z = 2x_1 + 3x_2 + x_3$

s.t. $2x_1 + x_2 - x_3 - S_1 = 3$

$x_1 + x_2 + x_3 - S_2 = 2$

$x_1, x_2, x_3, S_1, S_2 \geq 0$

All-slack/surplus basis is infeasible. So perform phase I computations by solving:

Minimize $Z = A$

s.t. $2x_1 + x_2 - x_3 - S_1 + A = 3$ ← "Most violated" Constraint. Set $A = 3$.

$x_1 + x_2 + x_3 - S_2 + A = 2$

$x_1, x_2, x_3, S_1, S_2, A \geq 0$

A will be in the starting basis, along with one other variable. An easy choice is to select one of the surplus variables, which means that

$$\underline{A = 3,\ S_2 = 1}$$

$B^1 = \{A, S_2\}$

$$\begin{aligned}
A &= 3 - 2x_1 - x_2 + x_3 + S_1 \\
S_2 &= -2 + A + x_1 + x_2 + x_3 \\
&= -2 + (3 - 2x_1 - x_2 + x_3 + S_1) + x_1 + x_2 + x_3 \\
&= 1 - x_1 + 0x_2 + 2x_3 + S_1 \\
\hline
Z &= A = 3 - 2x_1 - x_2 + x_3 + S_1
\end{aligned}$$

x_2 enters, A leaves. ← If x_1 were to enter, S_2 would leave, & thus A would stay in basis. Thus, x_2 is a more efficient choice.

$B^2 = \{x_2, S_2\}$

$$\begin{aligned}
x_2 &= 3 - 2x_1 + x_3 + S_1 - A \\
S_2 &= 1 - x_1 + 2x_3 + S_1 \\
\hline
Z &= 3 - 2x_1 - (3 - 2x_1 + x_3 + S_1 - A) + x_3 + S_1 \\
&= 0 + 0x_1 + 0x_3 + 0S_1 + A
\end{aligned}$$

Phase II

$B^0 = \{x_2, S_2\}$

$$\begin{aligned}
x_2 &= 3 - 2x_1 + x_3 + S_1 \\
S_2 &= 1 - x_1 + 2x_3 + S_1 \\
\hline
Z &= 2x_1 + 3(3 - 2x_1 + x_3 + S_1) + x_3 \\
&= 9 - 4x_1 + 4x_3 + 3S_1
\end{aligned}$$

x_1 enters, S_2 leaves.

$$x_1 = 1 + 2x_3 + S_1 - S_2$$
$$x_2 = 3 - 2(1 + 2x_3 + S_1 - S_2) + x_3 + S_1$$
$$= 1 - 3x_3 - 1S_1 + 2S_2$$

$B' = \{x_1, x_2\}$

$$Z = 9 - 4(1 + 2x_3 + S_1 - S_2) + 4x_3 + 3S_1$$
$$= 5 - 4x_3 - 1S_1 + 4S_2$$

x_3 enters, x_2 leaves (Note x_3 improves Z more than if S_1 enters)

$$x_3 = \frac{1}{3} - \frac{1}{3}x_2 - \frac{1}{3}S_1 + \frac{2}{3}S_2$$
$$x_1 = 1 + 2\left(\frac{1}{3} - \frac{1}{3}x_2 - \frac{1}{3}S_1 + \frac{2}{3}S_2\right) + S_1 - S_2$$
$$= \frac{5}{3} - \frac{2}{3}x_2 + \frac{1}{3}S_1 + \frac{1}{3}S_2$$

$$Z = 5 - 4\left(\frac{1}{3} - \frac{1}{3}x_2 - \frac{1}{3}S_1 + \frac{2}{3}S_2\right) - 1S_1 + 4S_2$$
$$= \frac{11}{3} + \frac{4}{3}x_2 + \frac{1}{3}S_1 + \frac{4}{3}S_2$$

$B^* = \{x_3, x_1\}$

$x_1^* = 5/3$, $x_2^* = 0$, $x_3^* = 1/3$

$Z^* = 11/3$