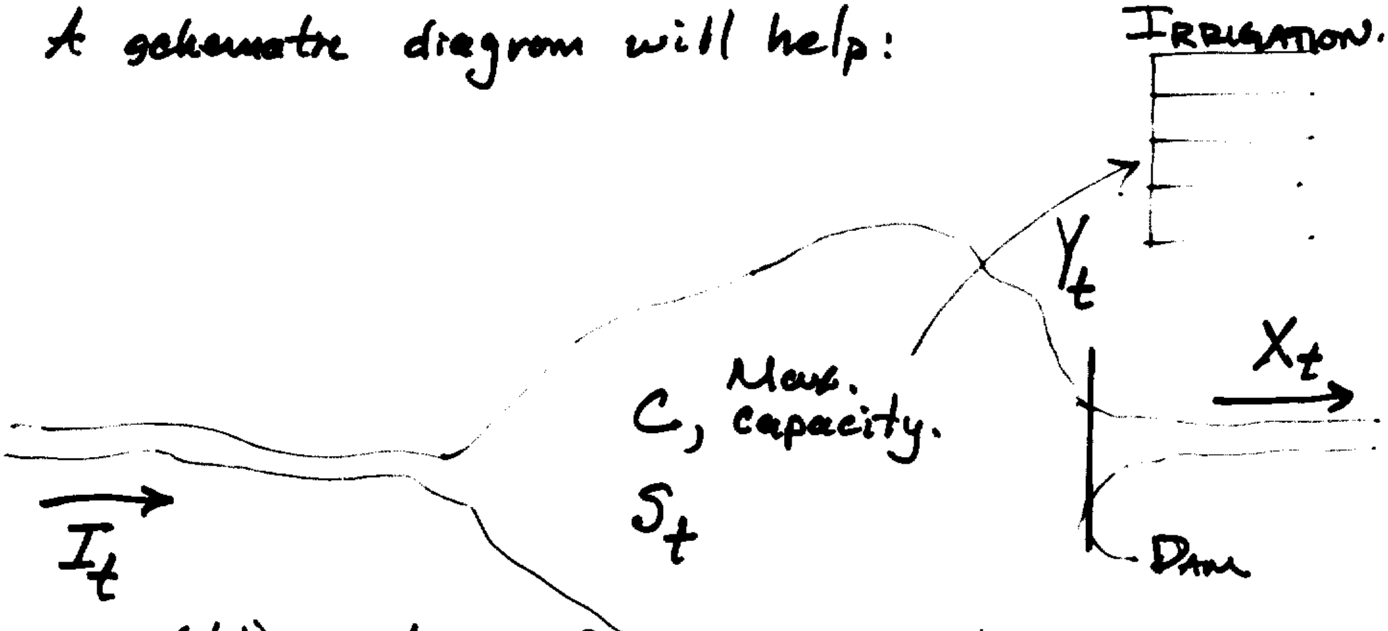
Multi-Objective Water Resources Planning.

Consider a surface water reservoir, which (typically) provide for drinking water supply, flood protection, irrigation water supply, recreation, and, perhaps, hydropower.

Think: What are the various ways in which these objectives conflict with each other?

What does it mean to have an "operating policy" for the reservoir? What are the decisions to be made and what are the inputs to those decisions?

A schematic diagram will help:



I_t (Vol) = Volume of water inflow to reservoir during month t. (Predicted input from forecast)

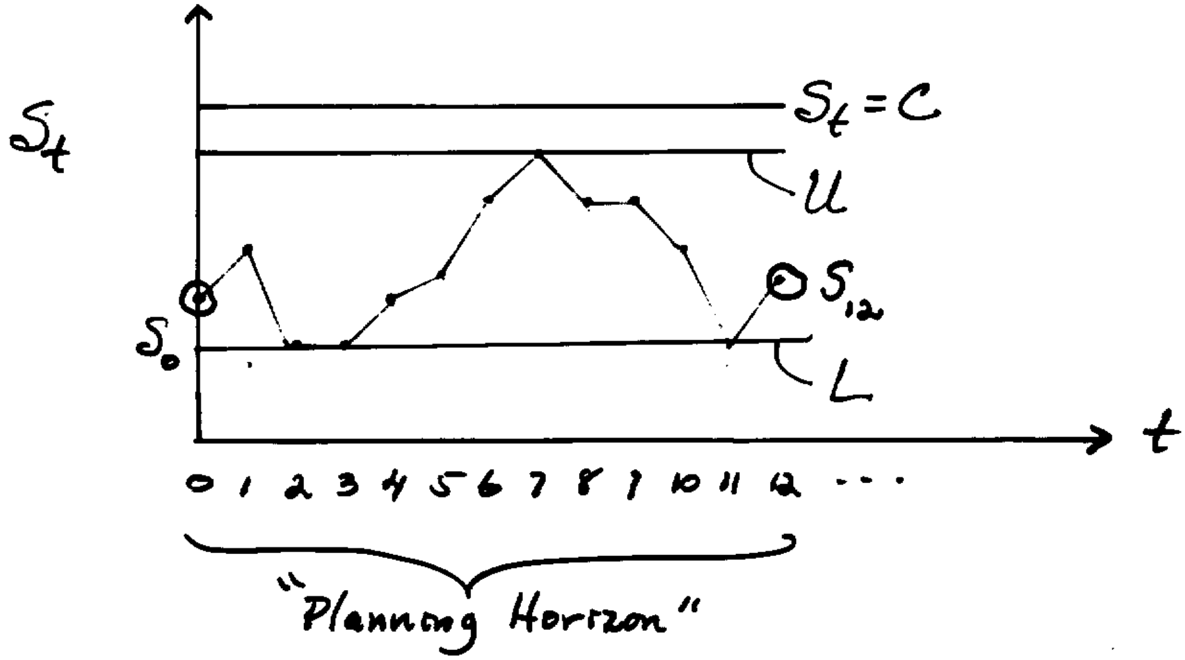
Y_t (Vol) = Volume of water to irrigation during month t. (to be determined)

X_t (Vol) = Volume of water to downstream use during month t. (to be determined)

S_t (Vol) = Volume of water stored in reservoir at end of month t. (to be determined)

$$\boxed{S_t = S_{t-1} - X_t - Y_t + I_t}$$

Fundamental Reservoir Storage equation.



The basic model (constraints):

$$S_t = S_{t-1} - X_t - Y_t + I_t \,, \quad t = 1, \ldots, 12 \qquad (1)$$

Forecasted.

Can not exceed physical capacity of Res. $\longrightarrow$

$$S_t \le C \,, \quad t = 1, \ldots, 12 \qquad (2)$$

$$S_{12} \ge S_0 \qquad (3)$$

OR, another suitable bound for water needed at beginning of next year.

Do not borrow water from the future! (i.e. leave things basically where you found them).

- How do we deal with uncertain forecasts of inflows?
  - → Obtain solution; implement; after 1st month re-forecast and re-solve model. ("recursive programming").
    Note: we have not yet discussed the model objectives, but we will.
    This is like using CPM iteratively during a construction project, adapting to conditions.

- Why do we consider a 12 month planning horizon? (or, why do we commonly do daily, weekly, yearly planning for many common tasks?)
  - → Because we believe that important features of the system dynamics will <u>repeat</u>, and hence can be forecasted using <u>history</u> as a guide.

Decreasing the fluctuation in Storage (One objective)

Several possible approaches:

Define $U \equiv$ the maximum storage value over the planning horizon
$L =$ the minimum storage volume over the planning horizon

And we may thus express the objective of decreasing the fluctuation in storage as

$$\text{Minimize } Z = U - L \qquad (4)$$

But how do we augment the constraints (1-3) so that U, L are as defined?

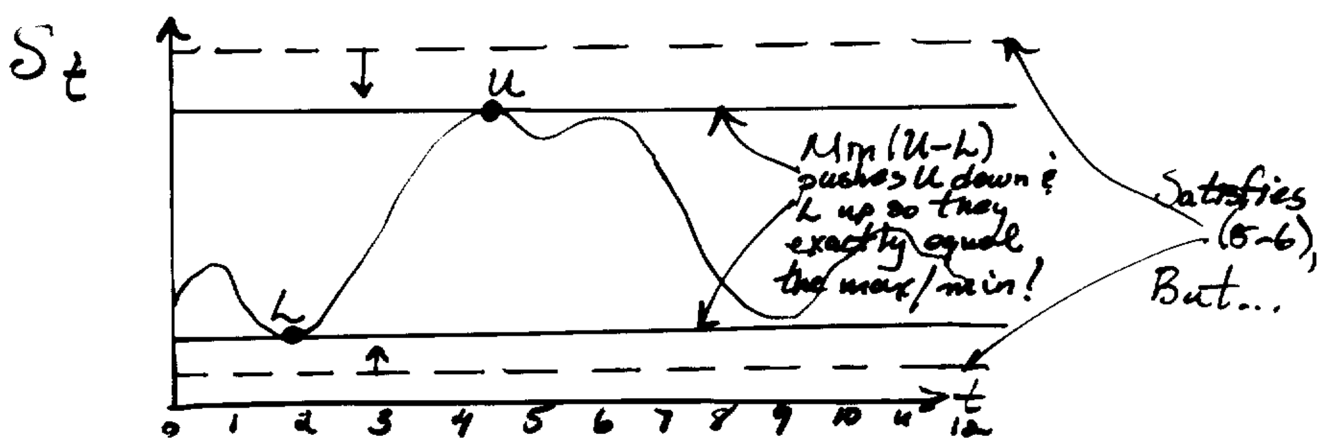
The following constraints prohibit U from being any less than the maximum storage over the planning horizon:

$$S_t \leq U \qquad t = 1, \ldots, 12 \qquad (5)$$

And similarly, for L:

$$S_t \geq L \qquad t = 1, \ldots, 12 \qquad (6)$$

But will these constraints, with the implied minimization in (4), always ensure U, L are exactly equal to the maximum & minimum, respectively?



So the final model is to minimize (4) s.t. (1–3) & (5–6), plus non-negativity:

$$\text{Minimize } Z = U - L$$

subject to:

$$S_t = S_{t-1} - X_t - Y_t + \boxed{I_t} \quad t = 1, \ldots, 12$$

$$S_t \le \boxed{C}, \quad t = 1, \ldots, 12$$

$$S_{12} \ge \boxed{S_0}$$

Parameters (values to be assumed)

$$S_t \le U, \quad t = 1, \ldots, 12$$

$$S_t \ge L, \quad t = 1, \ldots, 12$$

Variables.

$$U, L, S_1, \ldots, S_{12} \ge 0$$

$$X_1, \ldots, X_{12}, Y_1, \ldots, Y_{12} \ge 0$$

We could also model this objective in terms of the month to month change in storage, as the above formulation would allow most of the variation to occur during one period (month), which would not be desirable.

$$S_t - S_{t-1} \le 0.25\,(U - L) \qquad t = 1, \ldots, 12$$

← Limits the change over one period to less than 25% of max. change over entire horizon. Assuming $S_t \ge S_{t-1}$

ADD, TO ACCOUNT FOR STORAGE INCREASING OR DECREASING.

$$S_{t-1} - S_t \le 0.25\,(U - L) \qquad t = 1, \ldots, 12$$

Both constraints are needed.

Minimizing the deviation from an Irrigation Target

Again, many ways...

Define: T_t, $t=1,\ldots,12$ as the desired irrigation water usage during period t. This would be an optimal usage projection, for a particular crop mix.

Now, Let M = the maximum deviation from this target value.

We then wish to:

$$\text{Minimize } Z = M \tag{7}$$

And by similar reasoning to that above, we are required to add the following constraints to the basic model

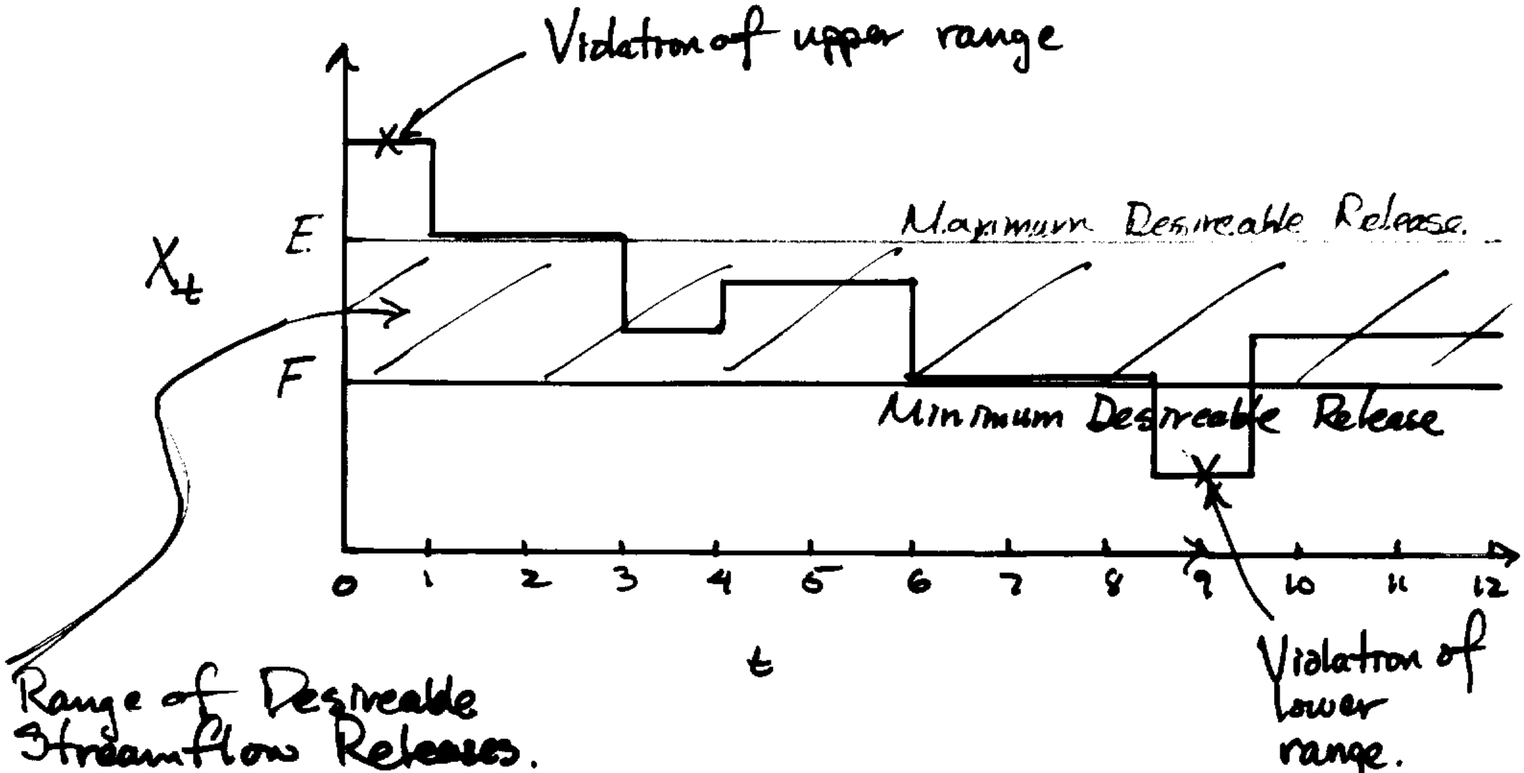
$$Y_t - T_t \le M \quad t=1,\ldots,12 \tag{8}$$

$$T_t - Y_t \le M \quad t=1,\ldots,12 \tag{9}$$

And the problem is to minimize (7), subject to (8-9) and (1-3).

Of course, we would normally use multi-objective methods to consider both objectives in the analysis, and to estimate the noninferior set (tradeoff between Irrigation & Storage fluctuation goals).

Minimizing the Sum of Deviations from Streamflow Release Target Interval.



- One way to express this objective is the sum of the violations (absolute, w/o regard to sign) over the planning horizon. We wish to minimize this sum (ideally it would equal zero).

- New Constraints:

(10) $$X_t - E = U_t^+ - U_t^- , \quad t = 1, 2, \ldots, 12$$

(11) $$\underbrace{X_t - F}_{\text{(signed) Deviation from lower target } F} = \underbrace{V_t^+}_{\text{Amt by which } X_t \text{ exceeds } F} - \underbrace{V_t^-}_{\text{Amt by which } X_t \text{ is less than } F}, \quad t = 1, 2, \ldots, 12$$

- And similarly for upper target constraints (10)

Q: What are the new variables, and parameters, we have introduced?

The objective function is to minimise the sum of violations of the upper (U_t^+) and lower (V_t^-) targets, over the planning horizon.

Release Target Objective:

$$(12) \qquad \text{Minimise. } Z = \sum_{t=1}^{12} U_t^+ + \sum_{t=1}^{12} V_t^-$$

Q: why do we "not care" about U_t^- & V_t^+ in the objective function?

Q: There are infinitely many values of U_t^+ / U_t' (or V_t^+ / V_t^-) that will satisfy (10) (or 11).

How do we know we are getting the correct values?

(Alternate question: Convince yourself that one of U_t^+ / U_t^- or V_t^+ / V_t^- will be zero at optimal policy.)