GRAPHICAL SOLUTION OF LINEAR PROGRAMMING PROBLEMS

- NOT A PRACTICAL TECHNIQUE (FOR REALISTIC PROBLEMS)
- ILLUSTRATE KEY CONCEPTS ABOUT LP SOLUTION.
  - DECISION SPACE
  - FEASIBLE REGION / SOLUTION
  - OPTIMAL SOLUTION
  - VERTEX / EXTREME POINT
- WILL LEARN ABOUT PRACTICAL SOLUTION TECHNIQUES (SIMPLEX ALGORITHM) LATER

Please recall the following fundamental concepts relating to plane geometry & the graphical depiction of linear functions:

Consider the following general linear function of two variables, x_1 & x_2:

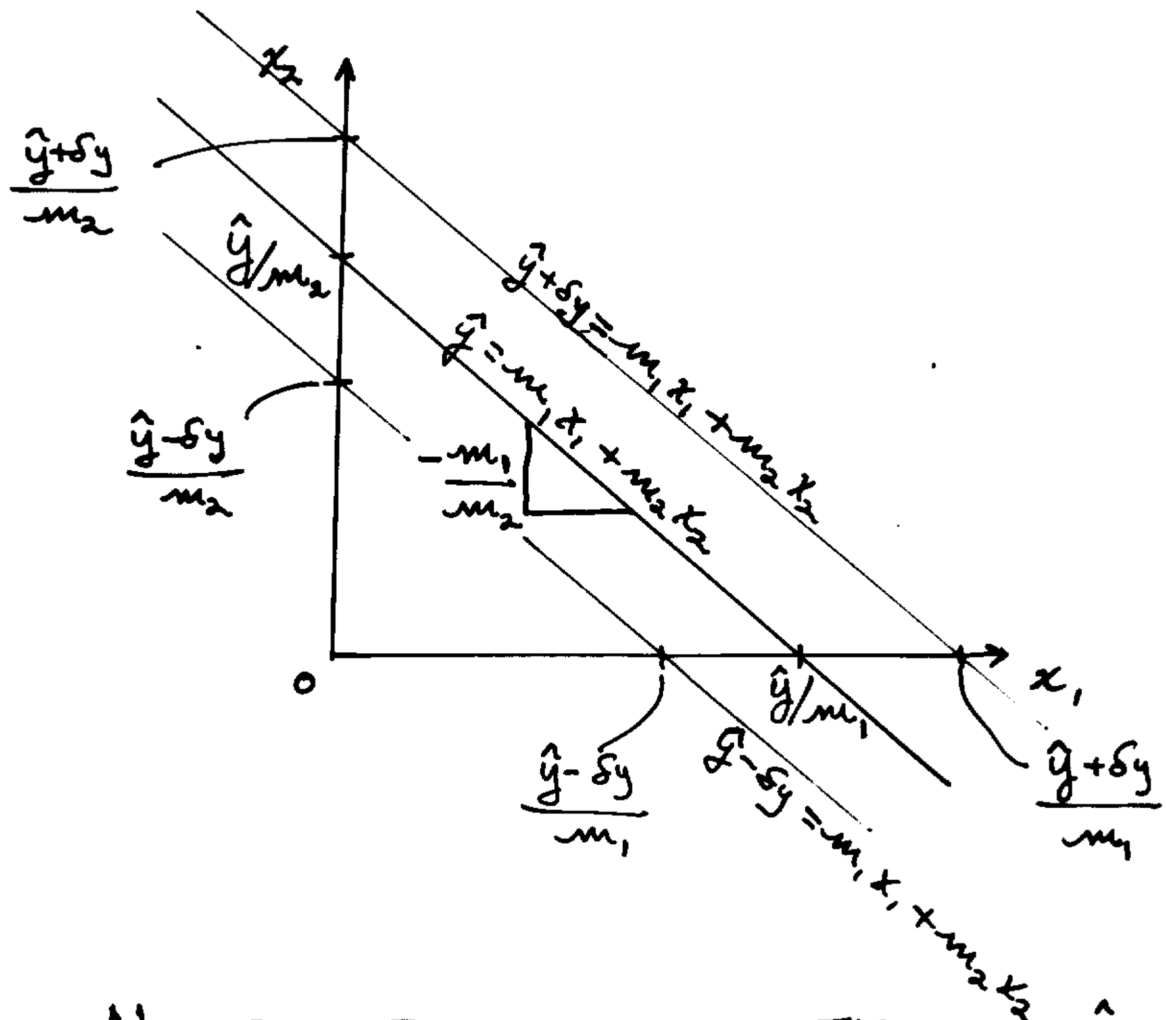
$$y = m_1 x_1 + m_2 x_2$$

where m_1, m_2 are constant parameters.

What does this look like when graphed?

This is the equation of a <u>plane</u> when depicted in the three-dimensional space of y, x_1, x_2.

We can also show this plane in the 2-D space of x_1, x_2, given constant values of $y = \hat{y}$



NOTE THAT TO <u>ONE SIDE</u> OF THE LINE $\hat{y} = m_1 x_1 + m_2 x_2$, the value of $y > \hat{y}$, while to the other side of the line we have $y < \hat{y}$.

EXAMPLE.

HOMEWOOD MASONRY.

Manufacture 2 products: HYDIT (concrete patch)
FILIT (brick mortar).

Summary of requirements for manufacture:

Resource	Hydit	Filit	Available
WABASH CLAY	$2m^3$/TON	$4m^3$/TON	28 m^3/WK
BLENDING TIME	5 hr/TON	5 hr/TON	50 hr/WK.
CURING VAT CAP.	8 TONS	6 TONS	—
PROFIT	\$ 140/TON	\$160/TON	—

IN WORDS!

MAXIMIZE TOTAL Weekly Profit

Subject to 4 production constraints:

- Total avail. supply of CLAY cannot be exceeded.
- Blending machine capacity can not be exceeded.
- Storage capacity for Hydit can not be exceeded.
- " " " Filit " " " " .

Mathematically:

Define: x_1 = # TONS OF HYDIT PRODUCED/WEEK.
x_2 = # TONS OF FILIT PRODUCED/WEEK.

$$\text{MAXIMIZE} \quad Z = \overset{(\$/\text{TON})}{140}\,\overset{(\text{TONS/WK})}{x_1} + 160\,x_2$$

$$\text{S.T.} \quad 2x_1 + 4x_2 \le 28 \qquad \text{(CLAY)}$$

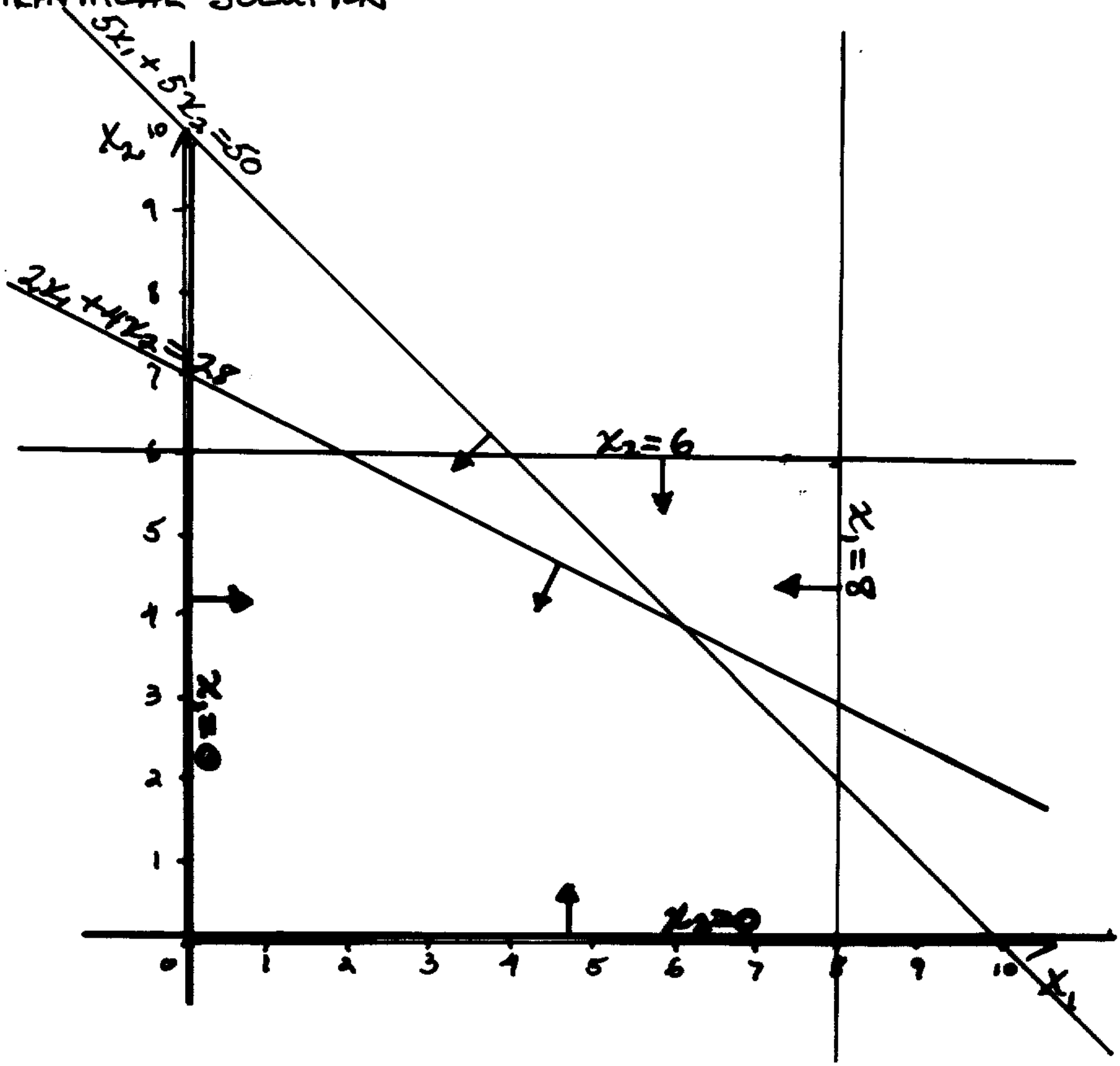
$$5x_1 + 5x_2 \le 50 \qquad \text{(MACHINE BLENDING)}$$

$$x_1 \le 8 \qquad \text{(Hydit CAP)}$$

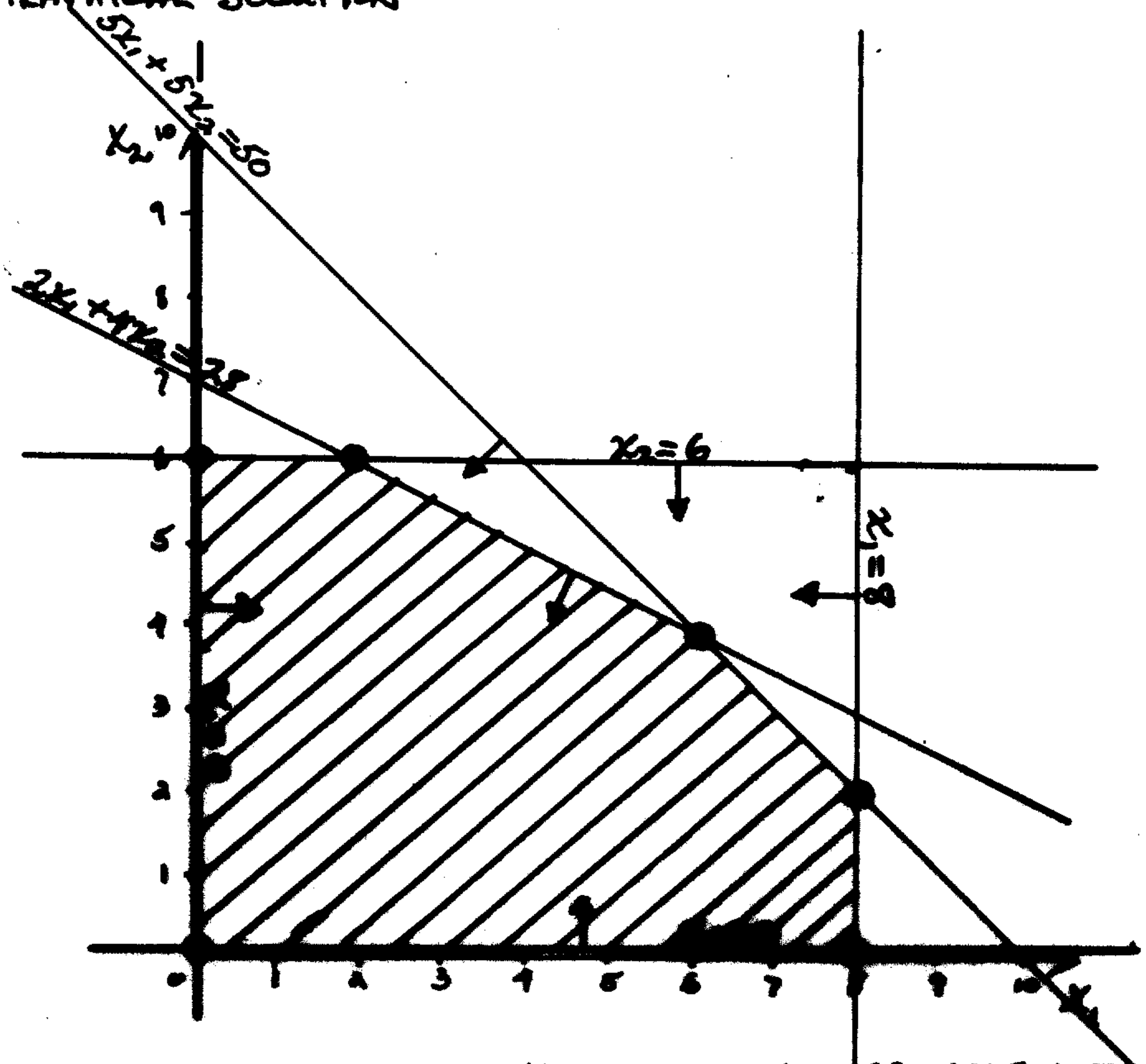
$$x_2 \le 6 \qquad \text{(Filit CAP).}$$

$$x_1,\ x_2 \ge 0 \qquad \text{(Non-negativity).}$$

GRAPHICAL SOLUTION



GRAPHICAL SOLUTION



/// - __FEASIBLE REGION__ (AN INFINITE NUMBER OF SOLUTIONS THAT __COULD BE__ IMPLEMENTED)

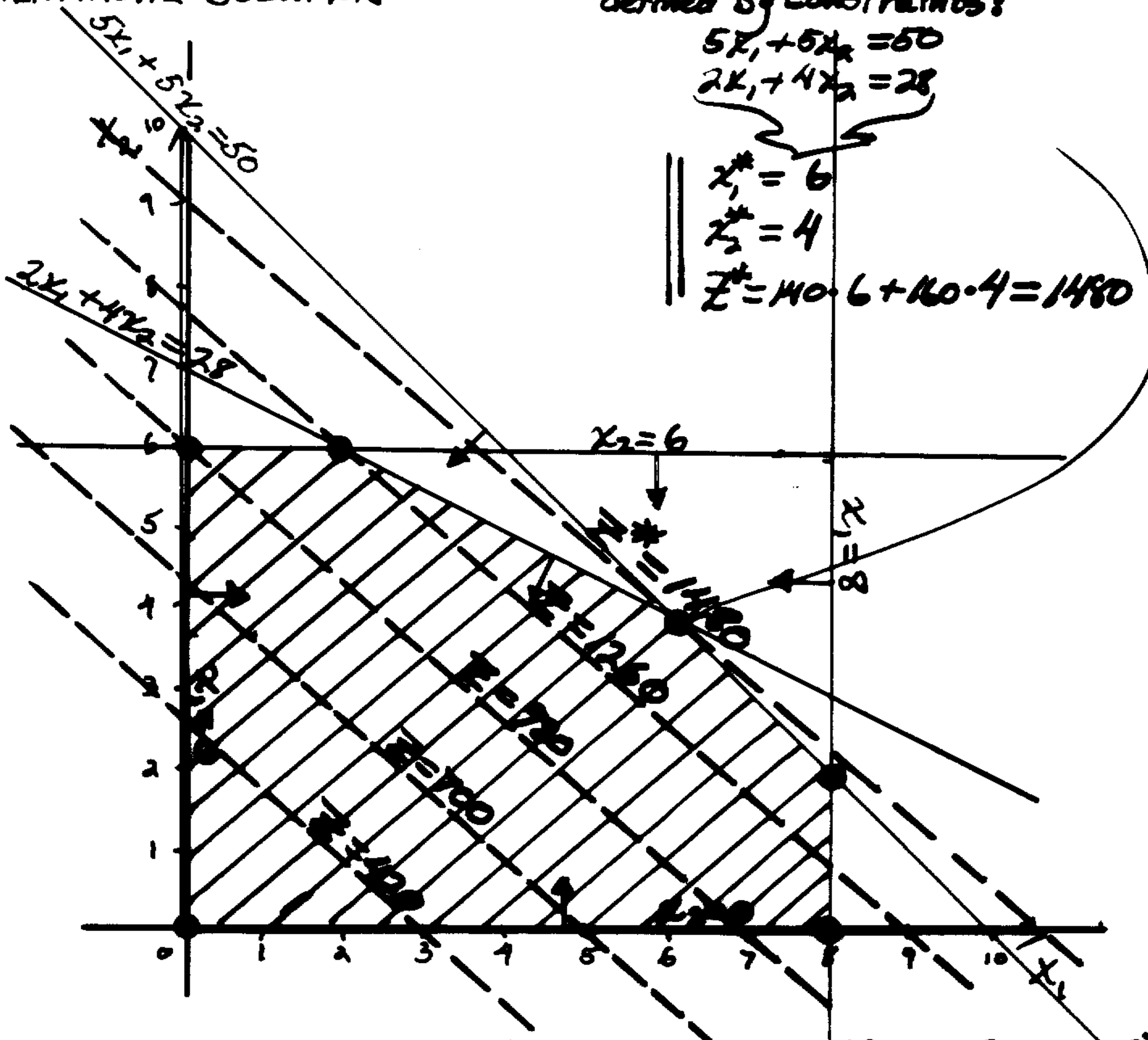
● - __EXTREME POINT__ OF F.R. (AT INTERSECTIONS OF CONSTRAINTS)

GRAPHICAL SOLUTION

Optimal Solution @ extreme pt defined by constraints:

$$5x_1+5x_2=50$$
$$2x_1+4x_2=28$$

$$x_1^*=6$$
$$x_2^*=4$$
$$Z^*=140\cdot 6+160\cdot 4=1480$$

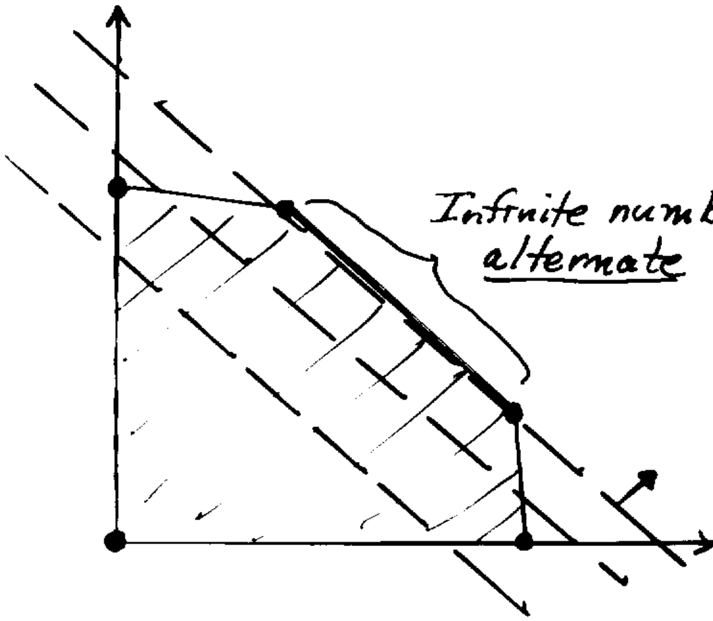


/// - FEASIBLE REGION (AN INFINITE NUMBER OF SOLUTIONS THAT COULD BE IMPLEMENTED)

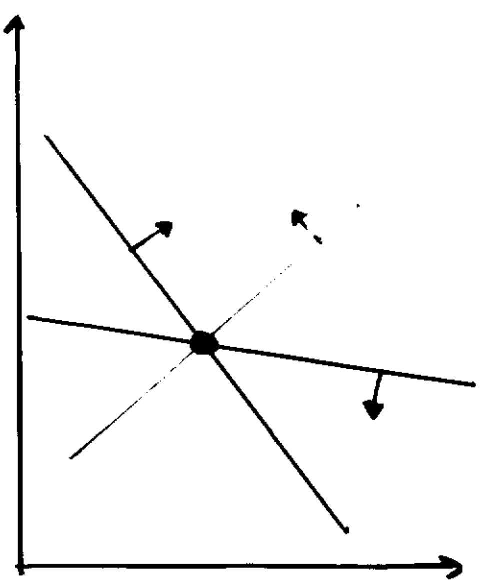
● - EXTREME POINT OF F.R. (AT INTERSECTIONS OF CONSTRAINTS)

* - OPTIMAL SOLUTION (A SINGLE FEASIBLE SOLUTION THAT MAXIMIZES OBJECTIVE FUNCTION)

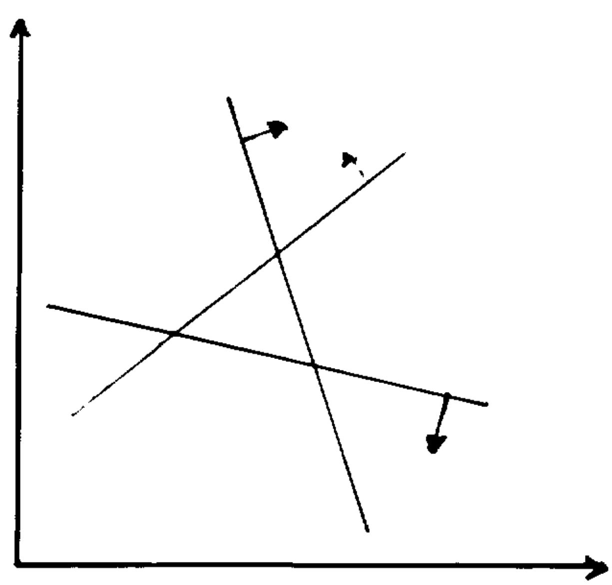
THERE EXIST SOME OTHER POSSIBILITIES FOR LP SOLUTIONS, TO BE AWARE OF:



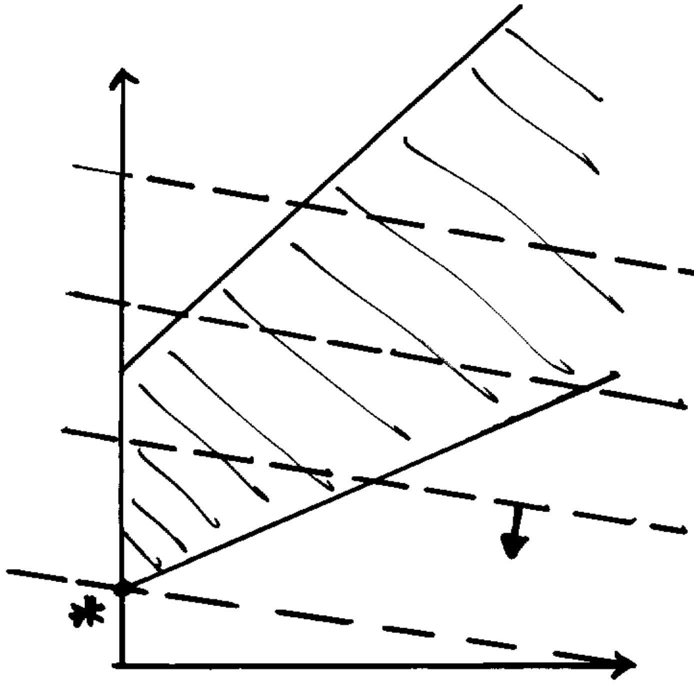
Occurs when objective function is strictly parallel to a constraint which defines a portion of feasible region (much more likely to be near optimal solutions)



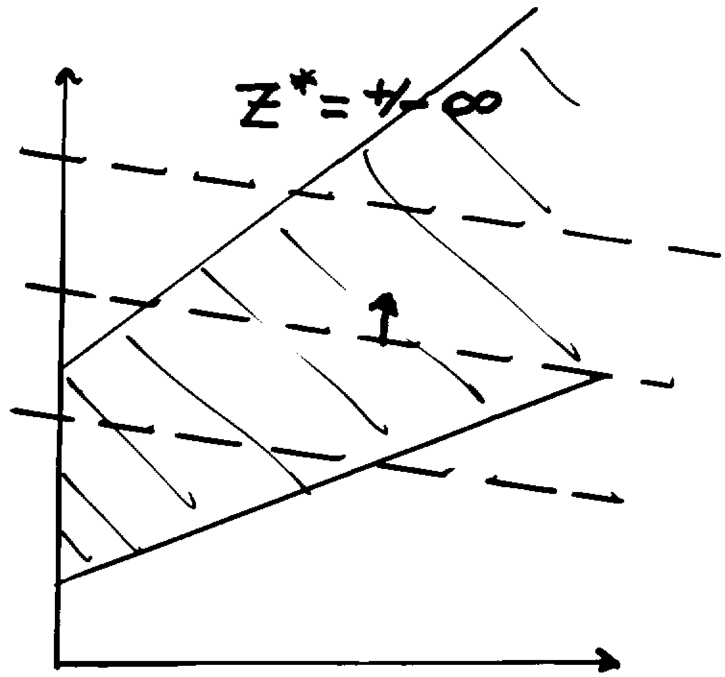
FEASIBLE REGION IS A SINGLE POINT



INFEASIBLE PROBLEM (NO FEASIBLE SOLUTIONS)



Unbounded feasible region / bounded solution objective value



Unbounded feasible region / unbounded solution objective value.