Homework 3
Polymer Physics
Due Monday September 11, 2022

The Mpemba effect is an observation that hot water freezes faster than cold water (https://en.wikipedia.org/wiki/Mpemba_effect) made by a Tanzanian (Tanganyikan at the time) schoolboy in 1963. Since that time there has been some interest in this subject. Holtzman R, Raz O *Landau Theory for the Mpemba Effect Through Phase Transitions* arXiv:2204.03995v1 [cond-mat.stat-mech] 8 Apr (2022) explore the use of Landau Theory to explain how the Mpemba effect could occur.

a) Equation (3) is the basis for the kinetics of phase formation by Holtzman where the rate of change of the order parameter depends on the negative of the gradient of the Landau free energy in the order parameter. ξ_i is "thermal noise". Holtzman explains why this term is important in the first partial paragraph, column 2 page 2. Use this description to explain what would happen if there were no thermal noise.

b) Holtzman explains that the Mpemba effect is not possible in systems with one order parameter in section III. B. This is true for the original Ising model for a ferromagnet (up or down spin arrows on a grid). For that system $x_1 = 0$ means a disordered system. When x_1 has a value there is order. Explain why such a system cannot display the Mpemba effect.

c) Appendix A on page 8, which is the supplemental section, describes a system with two order parameters, the antiferromagnet. Explain what the two order parameters are for this system and what physical and distinguishable features they describe. What is the difference between Glauber dynamics and Metropolis dynamics (https://en.wikipedia.org/wiki/Glauber_dynamics).

d) Figure 1b shows the salient reason that the Mpemba effect occurs, that is, why a hot system could show a faster transition to a new cold state compared to a cooler system. Explain, using Figure 1b the Mpemba effect.

e) Figure 3 summarizes Holtzman's results. What is the origin of the red line in Figure 3? How was the red line determined?