**Homogeneous** (assume a sphere of radius r)

$$ΔG\_{homo}=\frac{4π}{3}r^{3}ΔG\_{V}+4πr^{2}γ$$

$$\frac{dΔG\_{homo}}{dr}=0=4πr^{\*2}ΔG\_{V}+8πr^{\*}γ$$

$$r\_{homo}^{\*}=\frac{2γ}{ΔG\_{V}}$$

$ΔG\_{homo}^{\*}$= $\frac{4π8γ^{3}}{3ΔG\_{V}^{2}}+\frac{4π4γ^{3}}{ΔG\_{V}^{2}} $= $\frac{80πγ^{3}}{3ΔG\_{V}^{2}}=\frac{20πγr\_{homo}^{\*2}}{3}$

**Heterogeneous** (assume a disk of radius r, thickness t nucleating on the same material)

$$ΔG\_{hetero}=πtr^{2}ΔG\_{V}+2πrtγ$$

$$\frac{dΔG\_{hetero}}{dr}=0=2πtr^{\*}ΔG\_{V}+2πtγ$$

$$r\_{hetero}^{\*}=\frac{γ}{ΔG\_{V}}=\frac{r\_{homo}^{\*}}{2}$$

$ΔG\_{hetero}^{\*}$= $\frac{πtγ^{2}}{ΔG\_{V}}+\frac{2πtγ^{2}}{ΔG\_{V}} $= $\frac{3πtγ^{2}}{ΔG\_{V}}=\frac{9tΔG\_{V}ΔG\_{homo}^{\*}}{80γ}=\frac{3πtγr\_{homo}^{\*}}{2}=\frac{9t}{40r\_{homo}^{\*}}ΔG\_{homo}^{\*}$

$ΔG\_{hetero}^{\*}$= $\frac{9t}{80r\_{hetero}^{\*}}ΔG\_{homo}^{\*}$

$\frac{t}{r\_{hetero}^{\*}}\ll 1 and \frac{9}{80}<1$ so $ΔG\_{hetero}^{\*}\ll ΔG\_{homo}^{\*}$