

Introductory Chemical Engineering Thermodynamics

SOLUTIONS FOR HOMEWORK PROBLEMS: Chapter 10

(10.01) For a separations process it is necessary to determine the VLE compositions ...

Solution

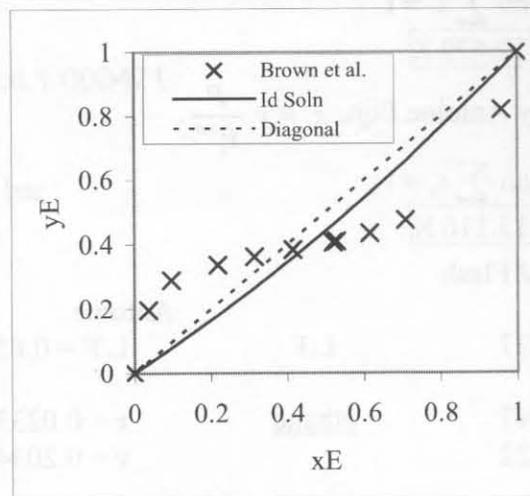
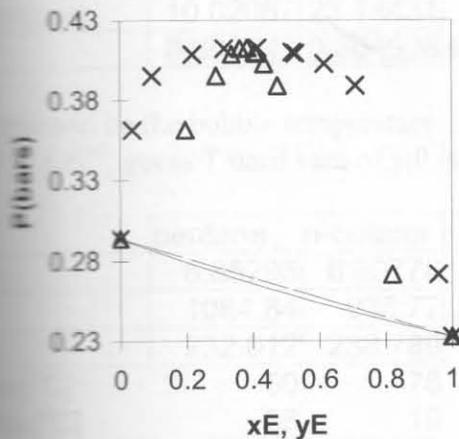
$$P = \sum x_i P_i^{\text{sat}} = .4723(.7569) + .5277(.0773) = .3983 \text{ therefore 12\% error in pressure.}$$

$$y_1 = .4723(.7569) / .3983 = .8975$$

(10.02) Benzene and ethanol form azeotropic mixtures.

$$\text{Solution } P^{is} = 0.2321 * X_e + 0.2939 * (1 - X_e) ; y^{is} = 0.2321 * x_e / P^{is}$$

x_E	0.0000	0.0374	0.0972	0.2183	0.3141	0.4150	0.5199	0.5284	0.6155	0.7087	0.9591	1.0000
y_E	0.0000	0.1965	0.2895	0.3370	0.3625	0.3842	0.4065	0.4101	0.4343	0.4751	0.8201	1.0000
P (bars)	0.2939	0.3613	0.3953	0.4088	0.4124	0.4128	0.4100	0.4093	0.4028	0.3891	0.2711	0.2320
y_E	0.2939	0.2916	0.2879	0.2804	0.2745	0.2683	0.2617	0.2612	0.2558	0.2501	0.2346	0.2320
P (bars)	0.0000	0.0298	0.0783	0.1807	0.2656	0.3591	0.4609	0.4695	0.5583	0.6577	0.9488	1.0000



(10.03) The following mixture of hydrocarbons ...

a) By short-cut vapor pressure eqn.

$$\frac{y_i}{x_i} = K_i \cong \frac{10^{\left[\frac{7}{3}(1 + \omega_i) \left(1 - \frac{1}{T_{2,i}} \right) \right]}}{P_{r,i}}$$

$$y_i = x_i K_i$$

$$\text{Find } T \text{ when } \sum_i y_i \cong \sum_i x_i K_i = 1$$

For given liquid composition, at $P = 0.5 \text{ MPa}$

$$\text{Bubble point temperature} = \boxed{293.376 \text{ K}}$$

OR

By Antoine Vapor Pressure Equation,

$$y_i = x_i \frac{P_i^{sat}}{P}; P_i^{sat} = 10^{\left[A_i - \frac{B_i}{T + C_i} \right]}$$

$$\text{Find } T \text{ when } \sum y_i = 1$$

A_i, B_i, C_i from tabulated in appendix or ACTCOEFF.xls

$$\text{bubble point temperature} = \boxed{294.68 \text{ K}}$$

b) Dew Point

$$\text{Short-cut method: } x_i = \frac{y_i}{K_i}$$

$$\text{Find } T \text{ when } \sum x_i = 1$$

$$T_{\text{dew.pt}} = \boxed{312.479 \text{ K}}$$

$$\text{Dew pt. By Antoine Eqn, } x_i = y_i \frac{P}{P_i^{sat}}$$

$$\text{Find } T \text{ when } \sum x_i = 1$$

$$T_{\text{dew.pt}} = \boxed{313.116 \text{ K}}$$

c) Isothermal Flash

Shortcut

$$L/F = 0.827$$

L/F

Antoine

$$L/F = 0.852$$

$$x = 0.02047$$

ethane

$$x = 0.02336$$

$$y = 0.19122$$

$$y = 0.20341$$

$$x = 0.08305$$

propane

$$x = 0.085735$$

$$y = 0.18100$$

$$y = 0.182122$$

$$x = 0.43173$$

n-butane

$$x = 0.42751$$

$$y = 0.24829$$

$$y = 0.24166$$

(10.05) A 50 mol% mixture of propane(1) and n-butane(2)...

a)

propane butane
 Psat(310.15K) 1.278795624 0.351249
 Ki Raoult's "=Psat/0.6 = =Psat/0.6 =
 2.132 0.585
 Work using FLSHR.xls but over-riding the Ki
 calculations.

Flash Conditions				
T(°C)	37	component	propane	butane
P(MPa)	0.6	K _i	2.132	0.585
L/F	2.37E-01	1-K _i	-1.132	0.415
		z _i	0.5	0.5
		D _i	0.00891	0.008416
		Objective Function		0.017326

$x_p = 0.5 / (2.132 + 0.237(1 - 2.132)) = 0.268$, $y_p = K_p * x_p = 2.132 * 0.268 = 0.572$

b)

fsatL(MPa) 1.045 0.321
 phiV at 0.6 MPa "=0.548/0.6= "=0.5133/0.6=
 0.913 0.856
 Ki (is) "=fsatL/phiV/P =
 1.907630522 0.625
 Work using FLSHR.xls but over-riding the Ki
 calculations.

Flash Conditions				
T(°C)	37	component	propane	butane
P(MPa)	0.6	K _i	1.908	0.625
L/F	2.17E-01	1-K _i	-0.908	0.375
		z _i	0.5	0.5
		D _i	0.008945	0.00838
		Objective Function		0.017325

$x_p = 0.5 / (1.908 + 0.217(1 - 1.908)) = 0.292$, $y_p = K_p * x_p = 1.908 * 0.292 = 0.558$

(10.06) A mixture of 55 mol% ethanol(1) and npropanol (2) is adiabatically flashed from 80°C and 2bar to 0.5 bar.

The stream is fed to an adiabatic flash drum. Calculate the outlet stream flow rates, temperatures, and compositions.(a) Use the path of Fig. 2.6(a). (b) Use the path of Fig. 2.6(c).

(a)Use the path of Fig. 2.6(a).

Heat of vaporization for 1-propanol from NIST Webbook, 47.5 kJ/mol

Other thermodynamic properties from Appendix E.

$0.05\text{MPa}(760\text{ mmHg})/0.101325\text{MPa} = 375\text{ mmHg}$

Solution found by modifying Ex10_02.m. Guesses of T are manual.

The answer is T = 68.45C, 341.6K. Comments are removed. See Ex10_02.m code for documentation.

```
function OBJEB = p10_06(T)
TF = 80; % Feed temp in C
Tref = 298.15; % Ref T in K
addpath ../Psat;
% ethanol - 1
% propanol - 2
[ names A B C ] = AntoineGet([ 3 5 ]);
Tb = [ 78.5 97.77 ] + 273.15; %K
Hvap = [ 38.58 47.5 ] * 1000; %J/mol
CpL = [ 281.6 -1.435 2.903E-03 0.;
        346.3 -1.749 3.552E-03 0. ];
CpV = [ 9.014 0.2141 -8.390E-05 1.373E-09;
```

```

    2.47    0.3325   -1.855E-04   4.296E-08];
P = 375; % mmHg
z = [0.55 0.45]; %overall composition
TFK = TF + 273.15; % Feed Temp in K
TK = T + 273.15; % Exit Temp in K
z = z/sum(z);
LOFguess = 0.5;
options=optimset('Display','iter'); % Option to display output
[LOF,fval,status] = fsolve(@calcObj,LOFguess,options); % Call optimizer
disp(sprintf('LOF = %g, OBJ = %g, status = %g', LOF, fval, status))
disp(sprintf('*****\nFlash Results. FeedT(C) = %g, T(C) = %g, P(mmHg) =
%g',TF,T,P))
disp(sprintf('1= %s, 2= %s',char(names(1)),char(names(2))))
format compact
LOF
z
x
y
K = y./x
HLi = [ Hcalc(CpL(1,:), Tref, TK) Hcalc(CpL(2,:),Tref, TK)];
HL = x*HLi';
HV1i = [ Hcalc(CpL(1,:), Tref, Tb(1)) Hcalc(CpL(2,:), Tref, Tb(2))];
HV2i = [ Hcalc(CpV(1,:), Tb(1), TK) Hcalc(CpV(2,:), Tb(2), TK)];
HV = y*[ HV1i + Hvap + HV2i ]';
HFi = [ Hcalc(CpL(1,:), Tref, TFK) Hcalc(CpL(2,:), Tref, TFK)];
HF = x*HFi';
OBJEB = ((1-LOF)*HV + LOF*HL - HF)/1000.;
function [obj] = calcObj(LOF)
    Psat = 10.^(A - B./(T + C));
    K = Psat/P;
    x = z./(K + LOF*(1-K));
    y = K.*x;
    obj = sum(x) - sum(y);
end
function [H] = Hcalc(Cp,T1,T2)
Cpc = Cp(1) + Cp(2)*T2 + Cp(3)*T2^2 + Cp(4)*T2^3;
H = Cp(1)*(T2 - T1) + Cp(2)/2*(T2^2-T1^2) + Cp(3)/3*(T2^3-T1^3) + ...
    Cp(4)/4*(T2^4-T1^4);
end % function Hcalc
end %function RaoultFL
-----
LOF = 0.964081, OBJ = 4.13698e-010, status = 1
*****

```

Flash Results. FeedT(C) = 80, T(C) = 68.45, P(mmHg) = 375							
1= ethanol, 2= 1-propanol							
LOF =	0.9641						
z =	0.5500	0.4500,	x =	0.5431	0.4569, y =	0.7358	0.2642
H =	1.3549	0.5781,	OBJEB =	-0.2227			

Using Antoine coefficients from Apx E.

At the entrance condition we have $H_{in} = HL = -278823$

T(K)	353.15	<HvapL>	<CpL>	<CpV>	HL	HV	H'
------	--------	---------	-------	-------	----	----	----

P (MPa)	0.2000		38902	75.32	29.51	-278823	-97574	-278823
q(vapor frac)	0.0000							
	KiIdSoln	z _i	x _i	y _i	Pisat(MPa)	Di	hVap	Hf
ethanol	0.542	0.550	0.5500	0.2980	0.1084	0.25	37787	-234950
propanol	0.254	0.450	0.4500	0.1144	0.0508	0.34	40263	-255200
At the bubble temperature ($\Sigma Di=0$) of 0.5bars we have HL = -280790, which is close.								
T (K)	341.17		<HvapL>	<CpL>	<CpV>	HL	HV	H'
P (MPa)	0.0500		39968	75.32	71.36	-280790	-237310	-280790
q(vapor frac)	0.0000							
	KiIdSoln	z _i	x _i	y _i	Pisat(MPa)	Di	hVap	Hf
ethanol	1.331	0.550	0.5500	0.7319	0.0665	-0.18	38878	-234950
propanol	0.596	0.450	0.4500	0.2681	0.0298	0.18	41300	-255200
Iterating on T and q to get H' = -278823 and $\Sigma Di = 0$.								
T (K)	341.33		<HvapL>	<CpL>	<CpV>	HL	HV	H'
P (MPa)	0.0500		39976	75.51	71.51	-280961	-237440	-278823
q(vapor frac)	0.0491						Dev=	0.0
	KiIdSoln	z _i	x _i	y _i	Pisat(MPa)	Di	hVap	Hf
Ethanol	1.339	0.550	0.5410	0.7245	0.0670	-0.18	38865	-234950
propanol	0.600	0.450	0.4590	0.2755	0.0300	0.18	41287	-255200

(10.07) An equimolar ternary mixture of acetone, n-butane, and ammonia ...

Solution:

(a) Known: P, x_1, x_2, x_3 . Unknown: $T, y_1, y_2, y_3, K_1, K_2, K_3$. Eqns: $y_i/x_i = K_i$ (3eqns), $\Sigma y_i = 1, K_i = P_i^{sat}/P$.

(b) Known: P, y_1, y_2, y_3 . Unknown: $T, x_1, x_2, x_3, K_1, K_2, K_3$. Eqns: $y_i/x_i = K_i$ (3eqns), $\Sigma x_i = 1, K_i = P_i^{sat}/P$.

(c) Known: $P, z_1, z_2, z_3, V/F$. Unknown: $T, x_1, x_2, x_3, y_1, y_2, y_3, K_1, K_2, K_3$.

Equations: $y_i/x_i = K_i, K_i = P_i^{sat}/P, x_i = z_i/[1+(V/F)(K_i-1)], \Sigma(x_i - y_i) = 0$

(d) Known: P, z_1, z_2, z_3, H^{in} . Unknown: $T, x_1, x_2, x_3, y_1, y_2, y_3, K_1, K_2, K_3, V/F$.

Equations: $y_i/x_i = K_i, K_i = P_i^{sat}/P, x_i = z_i/[1+(V/F)(K_i-1)], \Sigma(x_i - y_i) = 0,$

$H^{in} = H^{out} = (V/F)H^V + (1-V/F)H^L$.

Note: We write equations for K_i separately here to prepare for non-ideal solutions. Then the only change is the expression for K_i . For example, if we wrote $P = \Sigma x_i P_i^{sat}$ as the constraining equation for part (a), the connection to bubble point using an EOS model would not be straightforward. MEB instructors often teach just BUBP = $\Sigma x_i P_i^{sat}$ (like a definition) and then students perceive a contradiction in thermo with what they learned in MEB. By backing up one step, students can see that there is no contradiction, merely an added assumption that they should be prepared to alter in coming chapters.

(10.08) Tank A is rapidly half-filled with volatile hydrocarbon...

Tank B is 10 times as large and rapidly half-filled with the same hydrocarbon. Initially the gas space can be considered to be free of volatile organic and at the same pressure. The tanks are then closed. The tanks warm 20°C and the pressure in both tanks goes up. After warming, does one tank have a higher pressure than the other, or are the final pressures the same? Show your result with equations.

Solution: The final pressures are the same.

Known: T, z_1, z_2, z_3, V . Unknown: $P, x_1, x_2, x_3, y_1, y_2, y_3, K_1, K_2, K_3, V/F$.

Equations: $y_i/x_i = K_i, K_i = P_i^{sat}/P, x_i = z_i/[1+(V/F)(K_i-1)], \Sigma(x_i - y_i) = 0,$

$V = (V/F)V^V + (1-V/F)V^L$.

The n_1, n_2, n_3, V are different between the two systems, but the z_1, z_2, z_3, V are the same. Therefore the $x_1, x_2, x_3, y_1, y_2, y_3, K_1, K_2, K_3, V/F$ must be the same.

(10.09) Above a solvent's flash point temperature...

	methane	propane	pentane	hexane	ethanol	2-butanone	toluene	m-xylene	ethyl acetate
A	6.6438	6.80338	6.85296	6.87601	8.1122	7.280662	6.95087	7.00909	7.10179
B	395.74	804	1064.84	1171.17	1592.864	1434.201	1342.31	1462.266	1244.95
C	266.681	247.04	232.012	224.408	226.184	246.499	219.187	215.11	217.881
tMin[°C]	-182	-108	-50	-25	20	-6.5	-27	29	16
tMax[°C]	-158	-35.65	58	92	93	80	111	166	76
T(C)	-187.8	-104.5	-48.9	-21.7	12.7	-5.6	4.4	28.8	-4.5
Psat(mmHg)	42.3522	14.54983	10.9074	12.54264	27.8138	21.23863	8.85820	10.32726	18.50932
y=Psat/760	0.05573	0.019145	0.014352	0.016503	0.03660	0.027946	0.01166	0.013589	0.024354
LFL (Sax)	0.053	0.023	0.015	0.012	0.043	0.018	0.013	0.01	0.025

Sax, N.I., Dangerous Properties of Industrial Materials, 4th ed., Von Nostrand Reinhold, New York, 1975.

(10.10) Solvent vessels must be purged...

	hexane	1-butanol	chloroform	ethanol	toluene
A	6.87601	7.81028	6.95465	8.1122	6.95087
B	1171.17	1522.56	1170.966	1592.864	1342.31
C	224.408	191.95	226.232	226.184	219.187
tMin[°C]	-25	30	-10	20	-27
tMax[°C]	92	70	60	93	111
T(C)	22	22	22	22	22
Psat(mmHg)	132.7515	4.94141	172.7531	49.44508	24.29055
y _i = Psat/750	0.177002	0.006589	0.230337	0.065927	0.032387
Y _f	5.00E-04	1.00E-04	5.00E-05	1.00E-03	2.00E-04
t(min)	23.47723	16.75167	33.74111	16.75418	20.34883

(10.11) A pharmaceutical product is crystallized...

This is a semi-batch process. Crystals stay in dryer while gas passes through.

Volumetric flow rate of nitrogen $\dot{V} = 0.2 \text{ m}^3/\text{min}$

So molar flow rate \dot{n}_{N_2} would be $\frac{P\dot{V}}{RT} = (0.1 \cdot 0.2 \cdot 10^6) / (8.314 \cdot 323.15)$
 $= 7.444 \text{ moles/min}$

The saturation vapor pressure of the ethanol can be calculated using the Antoine equation

$\log_{10} P^{\text{sat}} = 8.1122 - \frac{1592.864}{226.184 + 50}$ which gives $P^{\text{sat}} = 221 \text{ mmHg} = 0.291 \text{ bar}$

Since the total pressure of the gases in the downstream of the drying process would be sum of the partial pressures of the EtOH and N₂ $P_{\text{total}} = P_{\text{EtOH}} + P_{\text{N}_2}$ and the ratio of partial pressure is the mole fraction y_i of the gas in the stream.

Hence we can write

$$\dot{n}_{EtOH} = \frac{y_{EtOH}}{y_{N_2}} \dot{n}_{N_2} = \dot{n}_{EtOH} = \frac{P_{EtOH}^{sat}(\text{bars})}{P - P_{EtOH}^{sat}(\text{bars})} \dot{n}_{N_2} = \frac{0.291}{1 - 0.291} 7.444 = 3.0523 \frac{\text{moles}}{\text{min}}$$

Basis: 100 kg wet crystals, 90 kg dry crystals, 10 kg EtOH

$$\text{Initial moles of EtOH in the product } n_{EtOH}^i = \frac{10000 \text{ grams}}{46 \text{ grams/mole}} = 217.39 \text{ moles}$$

$$\text{Final moles of EtOH in the product, } 0.001 = m_{EtOH} / (90000 + m_{EtOH}) \rightarrow m_{EtOH} = 90.1 \text{ g}$$

$$\text{Final moles of EtOH in the product } n_{EtOH}^f = \frac{90.1 \text{ grams}}{46 \text{ grams/mole}} = 1.96 \text{ moles}$$

$$\Delta n = 217.39 - 1.96 = 215.43 \text{ moles per 100kg feed}$$

$$\text{So the time for drying} = 215.43 \text{ mol} / 3.0523 (\text{mol/min}) = 70.57 \text{ minutes}$$

$$\text{So the flowrate of wet crystals into the dryer is } 100 \text{ kg} / 70.6 \text{ min} = 1.42 \text{ kg/min}$$

(10.12) Benzyl chloride is manufactured...

$$\text{Initial head volume of the tank : } V_{\text{head}}^i = 4.0 \text{ m}^3$$

$$\text{Final head volume of the tank : } V_{\text{head}}^f = 0.0 \text{ m}^3$$

Temperature: 303.15 K

Using ideal gas equation

$$n_{\text{head}}^i = PV_{\text{head}}^i / RT \text{ and } n_{\text{head}}^f = PV_{\text{head}}^f / RT$$

and the Raoult's law

$$y_i = x_i P_i^{sat} / P = x_i K_i$$

$$\text{we have } n_{\text{head}}^i = 0.1 * 4 / (8.314 * 10^{-6} * 303.15) = 158.7 \text{ moles, } n_{\text{head}}^f = 0 \text{ moles}$$

Using Antoine equation

$$P^{sat} = 10^{[6.95087 - 1342.31 / (30 + 219.19)]} = 37 \text{ mmHg for toluene}$$

$$P^{sat} = 10^{[7.597156 - 1961.47 / (30 + 236.51)]} = 1.727 \text{ mmHg for benzyl chloride}$$

$$K_{\text{toluene}} = 37 / 760 = 0.0487, \quad K_{\text{benzyl chloride}} = 1.727 / 760 = 0.00227$$

$$y_{\text{toluene}} = 0.5 * (0.0487) = 0.02435, \quad y_{\text{benzyl chloride}} = 0.5 * (0.00227) = 0.001136$$

$$\text{so } \Delta n_{\text{toluene}} = 0.02435 * (158.7 - 0) = 3.864 \text{ moles}$$

$$\& \Delta n_{\text{benzyl chloride}} = 0.001136 * (158.7 - 0) = 0.1803 \text{ moles}$$

(10.13) This problem explores emissions during heating...

(a)

$$T(\text{initial})(C) \quad 10$$

$$T(\text{final})(C) \quad 25$$

$$P(\text{mmHg}) \quad 760$$

	Hexane	Toluene
A	6.91058	6.95087
B	1189.64	1342.31
C	226.28	219.187

Psat(initial)	75.11144578	12.4176	mmHg
Psat(final)	150.0581956	28.4323	mmHg
x	0.5	0.5	Sum(yP)
yP(initial)	37.55572289	6.20879	43.76451
yP(final)	75.02909779	14.21616	89.24526
			y(nc)
y(initial)	0.049415425	0.008169	0.942415
y(final)	0.098722497	0.018705	0.882572

Headspace V	2.00E+05	cm3
Delta nnc	0.89738203	
Delta ni	0.073716616	0.013399
MW	86	92
Emission(g)	6.339628936	1.232729

(b)

T(initial)(C)	25
T(final)(C)	40
P(mmHg)	760

	Hexane	Toluene	
A	6.91058	6.95087	
B	1189.64	1342.31	
C	226.28	219.187	
Psat(initial)	150.0582	28.4323	mmHg
Psat(final)	277.3013	59.1487	mmHg
x	0.5	0.5	Sum(yP)
yP(initial)	75.0291	14.21616	89.24526
yP(final)	138.6507	29.57435	168.225
			y(nc)
y(initial)	0.098722	0.018705	0.882572
y(final)	0.182435	0.038914	0.778651

Headspace V	2.00E+05	cm3
Delta nnc	1.154498	
Delta ni	0.199817	0.041083
MW	86	92
emission(g)	17.18426	3.779614

(c)

T(initial)(C)	25
T(final)(C)	40
P(mmHg)	760

	Hexane	Toluene	
A	6.91058	6.95087	
B	1189.64	1342.31	
C	226.28	219.187	
Psat(initial)	150.0582	28.4323	mmHg
Psat(final)	277.3013	59.1487	mmHg
x	0.5	0.5	Sum(yP)
yP(initial)	75.0291	14.21616	89.24526
yP(final)	138.6507	29.57435	168.225

			y(nc)
y(initial)	0.098722	0.018705	0.882572
y(final)	0.182435	0.038914	0.778651

headspace		
V	5.00E+05	cm3
Delta nnc	2.886244	
Delta ni	0.499542	0.102707
MW	86	92
emission(g)	42.96065	9.449036

(d)

T(initial)(C)	25		
T(final)(C)	40		
P(mmHg)	760		
	Hexane	Toluene	
A	6.91058	6.95087	
B	1189.64	1342.31	
C	226.28	219.187	
Psat(initial)	150.0582	28.4323	mmHg
Psat(final)	277.3013	59.1487	mmHg
x	1	0	Sum(yP)
yP(initial)	150.0582	0	150.0582
yP(final)	277.3013	0	277.3013
			y(nc)
y(initial)	0.197445	0	0.802555
y(final)	0.36487	0	0.63513

headspace		
V	2.00E+05	cm3
Delta nnc	1.617461	
Delta ni	0.663565	0
MW	86	92
emission(g)	57.06655	0

(e) for toluene, (emission in b)/(emission in a) = 3.78/1.23 = 3.07

for hexane, (emission in b)/(emission in a) = 17.2/6.34 = 2.71

The ratio is larger for toluene because the heat of vaporization is larger and the vapor pressure increases faster with increasing temperature.

(10.14) The flash point is described in problems...

In each case the T for comp2= T for comp1. The T for comp1 is adjusted until $\sum y_i/LFL = 1$.

a			b			c		
	pentane	hexane		methanol	ethanol		benzene	toluene
A	6.85296	6.87601	A	8.08097	8.1122	A	6.87987	6.95087
B	1064.84	1171.17	B	1582.271	1592.864	B	1196.76	1342.31
C	232.012	224.408	C	239.726	226.184	C	219.161	219.187
tMin[°C]	-50	-25	tMin[°C]	15	20	tMin[°C]	8	-27
tMax[°C]	58	92	tMax[°C]	84	93	tMax[°C]	80	111
T(C)	-41.22	-41.22	T(C)	12.30	12.30	T(C)	-8.65878	-8.65878
P ^{sat} (mmHg)	18.70	3.04	P ^{sat} (mmHg)	63.5127	27.11	P ^{sat} (mmHg)	15.65	3.758
y _i =x _i P _s	0.0125	0.0020	y _i	0.042	0.018	y _i	0.01044	0.00251
t/750								
LFL	0.015	0.012	LFL	0.073	0.043	LFL	0.013	0.0127
y _i /LFL	0.831	0.169	y _i /LFL	0.58	0.42	y _i /LFL	0.803	0.197
Σy _i /LFL	1.000		Σy _i /LFL	1.000		Σy _i /LFL	1.000	

x ₁	x ₂	y ₁	y ₂	V ₁	V ₂
0	1	0	1	0	1
0.2	0.8	0.128	0.872	0.512	0.488
0.4	0.6	0.192	0.808	0.388	0.612
0.6	0.4	0.256	0.744	0.264	0.736
0.8	0.2	0.320	0.680	0.140	0.860
1	0	0	0	1	0

$\Delta V_{1,2} = V^L - V^V = 0.8 \text{ mol} \times (0.512 - 0.872) = -0.288 \text{ mol}$
 $\Delta V_{2,1} = V^V - V^L = 0.2 \text{ mol} \times (0.872 - 0.512) = 0.288 \text{ mol}$

