Introductory Chemical Engineering Thermodynamics

Homework Solutions

CHAPTER 4

(4.01) Extending Ex. 4.2 from solids to gases is straightforward for N2…

Adapting Ex. 4.2 for N2 then gives: $U^{ig} = 5RT/2 + \Delta U_{vib}$. We show in Eqn. 6.49 that $\Delta U_{vib} = \varepsilon_q/2 + \varepsilon_q/[\exp(-\beta\varepsilon_q)-1]$. For now, assume ΔU_{vib} as given.

a. Plot WebBook data for C_V^{ig} of N2 at 0.1 MPa and T=[150, 2000 K].

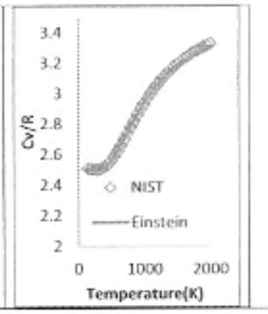
b. Derive an expression for C_V^{ig} based on above dissussion. Evaluate your expression at 1000K assuming $\varepsilon_q/k = 1500$K.

c. Regress an optimal ε_q/k of N2 and plot the calculated results with experimental data. Show the calculations as a curve with no points.

Solution: a. See table below.

b. Substituting and taking the derivative according to $C_v \equiv (\partial U/\partial T)_V$,

$Cv/R = 2.5 + (\beta\varepsilon_q)^2\exp(-\beta\varepsilon_q)/(\exp(-\beta\varepsilon_q) - 1)^2$ (cf. Eqn. 6.50)



At $\beta\varepsilon_q = \varepsilon_q/kT = 1500/1000$, $Cv/R = 2.5 + 1.5\text{^}2*\exp(-1.5)/(\exp(-1.5)-1)\text{^}2 = 3.332$

c. Solver: Set rmse=sqrt(sumxmy2(calc,expt)/count(expt)) = min

by changing ε_q/k where $\beta\varepsilon_q = \varepsilon_q/kT$. => **$\varepsilon_q/k$ = 3261K. rmse = 0.00975**

T(K)	150	300	500	600	700	800	1000	1200	1400	1600	1800	2000
a. Cv/R(NIST)	2.51	2.50	2.56	2.62	2.70	2.78	2.93	3.06	3.15	3.23	3.28	3.33
$\beta\varepsilon_q$	21.74	10.87	6.52	5.44	4.66	4.08	3.26	2.72	2.33	2.04	1.81	1.63
c. Cv/R(calc)	2.50	2.50	2.56	2.63	2.71	2.79	2.94	3.06	3.15	3.22	3.27	3.31

(4.02) An ideal gas with temperature-independent Cp =7/2 R …

Solution :

T_i= 15 C = 288 K $V_1 = 8.314*288/0.1 = 23944.34$ cm^3/mol

T_2 = 30 C = 303 K $V_2 = V_1* T_2/ T_1 = 255191.42$ cm^3/mol

$n = P_1\underline{V}_1/RT_1 = 0.1013*60E6/8.314/288.15 = 2537$ mol

Energy Balance: dU = (dQ + dW) = dQ - P dV

S- Balance (closed system): dS = C_P dT/T - RdP/P

$$\Delta \underline{S}_{gas}^{id} = n\left[C_P \ln\frac{T^f}{T^i} - R\ln\frac{P^f}{P^i}\right]$$

$$= 2538.389\times(7/2)\times 8.314 \ln(303/288) = 3750.2767 \text{ J/K}$$

$$\Delta S_{res} = \frac{Q_{res}^{rev}}{T_{res}} = \frac{-Q_{gas}}{T_{res}} = \frac{-nC_P(T^f - T^i)}{T_{res}}$$

$$= -\ 3430.2438 \text{ J/K}$$

$\Delta S_{univ} = 320.033$ J/K (because of heat transfer)

(4.03) Steam undergoes a state change from 450 C and 3.5 MPa to 150°C and 0.3MPa. Determine ΔH and ΔS for (a) steam tables (b) IG assumption.

a) State	Initial	Final
Temperature	450 °C	150 °C
Pressure	3.5 MPa	0.3 MPa

From steam table we find these values are in superheated phase

Enthalpy	3338.0 kJ/kg	2761.2 kJ/kg
Entropy	7.0074 kJ/kgK	7.0791 kJ/kgK

so $\Delta H = -576.8$ kJ/kg; $\Delta S = 0.0717$ kJ/kgK

b) Using ideal gas assumption,

C_P of water = 4.041*R = 33.6KJ/kmolK = 1.87 KJ/kgK

$\Delta H = mC_P\Delta T = 1 * 1.87*(150 - 450) = -560$ KJ/kg

$\Delta S = [C_P \ln(T^F/T^I) - R\ln(P^F/P^I)] = [33.6 \ln(423/723) - R*\ln(0.3/3.5)]$

$\Delta S = 2.41$ kJ/kmol-K = 0.134 kJ/kg-K

Note: IG assumption is not too bad here because the temperature is high when the pressure is high (T_c = 647-273=374C), and vice versa.

(4.04) The following problems involve one mole of an ideal monatomic gas...

a) Energy balance: closed system with one mole of ideal gas

$$\Delta U = \cancel{W_S} + \cancel{W_{EC}} + Q$$

$$\Delta U = Q \qquad \Delta U = 5 \text{ kJ}$$

$$Q = C_V\Delta T,\ C_V = C_P - R$$

$$C_V = \frac{3}{2}R = 1.5(8.314 \text{ J/mol K}) = 12.47 \text{ J/mol}$$

$$\Delta T = \frac{\Delta U}{C_V} = \frac{5000}{12.47} = 400.96 \text{ K}$$

$$\Delta S_V = \int \frac{dQ}{T} = \int \frac{C_V dT}{T} = C_V \ln\frac{T_2}{T_1} = 12.47 \ln\frac{298.15 + 400.96}{298.15}$$

$$\Delta S_V = 10.63 \text{ J/mol K}$$

$$PV = RT$$
$$\frac{V}{R} = \frac{T_1}{P_1} = \frac{T_2}{P_2} = \frac{298.15}{0.5} = \frac{699.11}{P_2}$$
$$P_2 = 1.172 \text{ MPa}$$
$$\Delta P = 1.172 - 0.5 = 0.67 \text{ MPa}$$

b) PV = nRT, n = 1 mole
$V^i = RT^i / P^i = 83.143\ (\text{cm}^3\text{-bar/mole-K}) * 298.15\text{ K} / 5\text{ bar} = 4,957.82\text{ cm}^3$

Energy balance: still a closed system with one mole of ideal gas

$$\Delta U = \cancel{W_S} + W_{EC} + \cancel{Q} \qquad \text{(volume is changing)}$$

Need to find V^f, use the entropy balance

Entropy balance
Assume isentropic (adiabatic and reversible)

$$dnS = n(\underset{\text{no flow}}{S^{in} - \cancel{S^{out}}}) + \underset{\text{adiabatic}}{\cancel{dQ}/T} + \underset{\text{reversible}}{\cancel{Sgen}}$$

$$\mathbf{\Delta S = 0} = C_V * \ln(T^f / T^i) + R*\ln(V^f / V^i)$$

Rearrange and solve for V^f,
$$V^f = V^i * (T^f / T^i)^{-C_V/R}$$
$$= 4,957.82\text{ cm}^3 * (699.08 / 298.15)^{-1.5} = 1,380.87\text{ cm}^3$$
$\mathbf{\Delta V} = V^f - V^i = 1,380.87 - 4,957.82 = \mathbf{-3576.95}\text{ cm}^3$ (volume has decreased)

$P^f = nRT^f / V^f = 1\text{ mole} * 8.3143\text{ cm}^3\text{-MPa/mole-K} * (699.08\text{ K} / 1380.87\text{ cm}^3)$
$P^f = 4.21$ MPa
$\mathbf{\Delta P} = 4.21 - 0.5 = \mathbf{3.71}$ MPa

From the energy balance,
$\Delta U = W_{EC} = C_V * (T^f - T^i)$
Since it's a closed system and the temperature rise is the same as in case (a), ΔU is the same, $\mathbf{\Delta U} = \mathbf{5}$ kJ

c) $W_S = +5$ kJ

d) 0.7(55 watts)(1J / 1 watt sec) = 38.5 J/sec
$$\dot{U} = \dot{W}_S = C_V \dot{T}$$
$$\dot{T} = \frac{\dot{W}_S}{C_V} = \frac{38.5}{12.47} = 3.1 \text{ K/sec}$$
$$\dot{U} = 38.5 \text{ J/sec}$$

$$\dot{S} = \dot{S}_{gen} = C_V \frac{dT}{Tdt} = \frac{C_V}{T}(3.1 \text{ K/sec})$$
$$= \frac{12.48}{298}(3.1) = 0.13 \text{ J/mol K sec}$$

e) $\dot{U} = 0 = \dot{Q} + \dot{W}_{EC}$

Must cool $\dot{Q} = -\dot{W}_{EC}$

$$\dot{W}_{EC} = -P\frac{dV}{dt} = -\frac{RT}{V}\frac{dV}{dt}$$

$V = 4957.6 - 2t$ where t in sec

$$\dot{Q} = \frac{8.314(298.15)}{4957.6 - 2t}$$

Total heat transfer,

$$\Delta U = Q + W_{EC} = 0$$
$$Q = -W_{EC} = RT\ln\frac{V_2}{V_1} = -8.314(298.15)\ln 2 = \text{-1718 J}$$

(4.05) When a compressed gas storage tank fails, the resulting explosion…

a) $n\Delta U = nCv(T^f - T^i) = \underline{W}_{EC}$

n and the initial temperature are not given, but P and $\underline{V}$ are given.

$P\underline{V}/R = nT$ and $V^f = V^i(P^i/P^f)^{Cv/Cp}$, since the system is considered closed (no mixing with surrounding air, $\underline{V}^f = \underline{V}^i(P^i/P^f)^{Cv/Cp}$.

$\underline{V}^f = 2.5(15/1)^{5/7} = 17.3 \text{ m}^3$, $(nT)^f = 0.1 \text{ MPa}(17.3\text{E}6\text{cm}^3)/8.314 = 2.081\text{E}5$ molK

$(nT)^i = 1.5 \text{ MPa}(2.5\text{E}6\text{cm}^3)/8.314 = 4.510\text{E}5$ molK

$n\Delta U = 5*8.314/2*(2.081\text{E}5 - 4.510\text{E}5) = \underline{-5.049 \text{ E}6 \text{ J}} = \underline{W}_{EC}$

b) 5.049E6J/(4.5E6 J/[kg TNT]) = 1.12 kg TNT

(4.06) work problem 4.5, but consider a steam boiler that fails…

a) from sat'd P table, $V^L = 0.001253$ $V^V = 0.0498$ $U^L = 1082.48$ $U^V = 2601.72$
$S^L = 2.7968$ $S^V = 6.0696$
From problem statement, $m^L V^L = 0.250 \text{ m}^3/2$, $m^L = 99.76$ kg liquid
$m^V V^V = 0.250 \text{ m}^3/2$, $m^V = 2.51$ kg vapor, $m_{total} = 102.3$ kg.
$\underline{U}^i = m^L U^L + m^V U^V = 99.76(1082.48) + 2.51(2601.72) = 1.145\text{E}5$ kJ
The expansion will be isentropic,
$\underline{S}^i = m^L S^L + m^V S^V = 99.76(2.7968) + 2.51(6.0696) = 294.24$ kJ/K
At 0.1 MPa, $S^V = 7.359$, and $(99.76 + 2.51)(7.359) = 752.6 \text{ kJ/K} > \underline{S}^i$, therefore system will remain two-phase. At 0.1 MPa,
$V^L = 0.001043$ $V^V = 1.694$ $U^L = 417.4$ $U^V = 2505.6$
$S^L = 1.303$ $S^V = 7.359$, using S-balance to find quality, $\underline{S}^f = \underline{S}^i$,
$294.24 = (99.76 + 2.51)(1.303 + q6.056)$, $q = 0.26$, so quality goes up considerably. $\underline{U}^f =$

$m^LU^L + m^VU^V = 102.3(417.4 + 0.26(2088.2)) =$
$9.824E4$ kJ, $\Delta\underline{U} = 9.824E4 - 1.145E5 = -1.63E4$ kJ = $\underline{W_{EC}}$

b) (1.63E4 kJ)/(4.5 MJ/[kg TNT]) = 3.6 kg TNT
exercise: rework the problem with the initial boiler 85% vapor and 15% liquid. What do you find?

(4.07) Steam expands from 600K and 1MPa into a volume that is double. Compute Tf, Pf, ΔS based on (a) IG with Cp/R = 3.5 (or 4.041) (b) steam.

(a) an ideal gas
E-bal: $dU = Q + W = 0 + 0 = 0$
S-bal: $dS = Qdt/T + S_{gen}dt = S_{gen}dt$
a) For ideal gas: $dU = C_v^{ig}\, dT = 0$ (volume isn't constant, but no energy to spread ideal gas molecules.)
Constant temperature ideal gas doubling in volume. (Same as Eqn. 4.22)
$RT\ln(V/V^i) = T\, S_{gen} \Rightarrow S_{gen} = R\ln(2) = 5.763$ J/mole-K (Note: Cp/R irrelevant)
b) For steam, we must use steam tables (alt. STEAM.xls, below). (327°C)
$dU = 0 \Rightarrow U_f = U_i = 2837.9$ J/g ; $V_f = 2V_i = 2(0.2712) = 0.5424$; $S_i = 7.2209$
Guess $P = 0.5$ MPa. Interpolation $\Rightarrow T@V{=}0.5424 = 320.8$°C
$\Rightarrow U_f = 2836.4$ close enough to $2837.9 \Rightarrow S_f = 7.5334$
$\Rightarrow \Delta S = (7.5334 - 7.2209)$J/g-K*18g/mole = 5.625 J/mole-K

Alt. solution using STEAM.xls: Tf = 594.9, Sf=7.5377 ; Pf=0.5009
$\Rightarrow \Delta S = (7.5377 - 7.2209)$J/g-K*18g/mole = 5.702 J/mole-K

(4.08) An isolated chamber is divided into two equal halves…

a) Ideal Gas
$dU = dQ + dW_{EC} = 0$ since $dQ = dW_{EC} = 0$ ($\therefore dU = 0$ and T = constant)

$$\Delta S_{gas} = C_P \ln\frac{T^f}{T^i} - R\ln\frac{1}{2} = 5.7628 \text{ J/mol K}$$

b) Steam
$dU = dQ + dW_{EC} = 0$ since $dQ = dW_{EC} = 0$
$U^i = U^f$ and $V^f = 2V^i$
at 25 MPa and 650 K (377 C), extrapolating from 400 and 450C,
$U^i = 2294$ kJ/kg, $S^i = 4.8935$ kJ/kg-K
$V^i = 0.00453$ m^3/kg
$\therefore V^f = 0.00906$ m^3/kg $U^f = 2294$ kJ/kg
Scanning tables, answer must be in 2-phase region.

$$\frac{U^f - U^L}{\Delta U^{vap}} \equiv q \equiv \frac{V^f - V^L}{\Delta V^{vap}}$$

Let us find the solution by guessing T, calculating q from V, then checking the value of q from U by an objective function ($q_{\text{from V}} - q_{\text{from U}}$).
Iterating to solve using a spreadsheet is quickest since the formulas can be applied quickly and the results are immediately tabulated,

T(C)	P(MPa)	V^L	V^V	U^L	U^V	q from V	q from U	obj func	S^L	S^V	on V
330	12.858	0.001561	0.013	1505.8	2499.15	0.655564	0.793477	-0.13791	3.5518	5.4422	4.791079
340	14.6	0.001638	0.0108	1570.62	2464.44	0.810085	0.809313	0.000772	3.6601	5.3356	5.017398

We have spanned the state, the final state is close to 340 C and 14.6 MPa.
Assuming linear interpolation with respect to T, the linear interpolation formulae can be entered
and we can find quickly using solver or Goal seek.

T(C)	P(MPa)	V^L	V^V	U^L	U^V	q from V	q from U	obj func	S^L	S^V	on V
339.952	14.59165	0.001638	0.010811	1570.309	2464.606	0.809161	0.809228	-6.8E-05	3.659581	5.336111	5.016164

T = 340 °C, P = 14.59 MPa
$q = 81\ \%$
$S^f = 5.0162$ kJ/kg-K
$\Delta S_{gas} = S^f - S^i =$ 0.1227 J/g-K*18 g/mol = 2.2086 J/mol-K
(the entropy change is less than if it had remained as one-phase because liquefaction causes the entropy to decrease).

(4.09) Airplanes are launched from aircraft carriers by means of a steam catapult.

Solution : It cannot generate more than the adiabatic reversible result. But in principle it could generate at most the adiabatic reversible result.

Energy Balance : d(mU)= Q + W = W
Entropy balance : Δ S = 0

State 1 : $U_1 = 2880.7$
$S_1 = 6.1434$
$V_1 = 18.5\ \text{cm}^3/\text{g}$

State 2 : Sat liq U =604.22 S= 1.7765
Sat Vap U =2553.1 S = 6.8955
$V^{satL} = 1.08\ \text{cm}^3/\text{g}$, $V^{satV} = 462\ \text{cm}^3/\text{g}$

Therefore, work done by gas
q = (6.1434 – 1.7765)/(6.8955 – 1.7765) = 0.853
$U_2 = 0.853 * 2553.1 + 0.147*604.2 = 2266.6$
$W_{gas} = 2266.6 - 2880.7 = -614.1$ kJ/kg
$\underline{W}_{gas} = 270 *(-614.1) = -165{,}800$ kJ

Some of the work is done on the atmosphere, need $P_{atm}\Delta\underline{V}$
$V^f = 1.085 + 0.853(461) = 394\ \text{cm}^3/\text{g}$
$\underline{W}_{EC,\ atm} = -P_{atm}(\underline{V}^f - \underline{V}^i) = -0.1\text{MPa}*(270\text{E}3\text{g})(394-185) = 10{,}139$ kJ

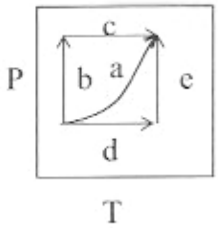
Net work (ignoring acceleration of piston -- mass not given) = -165,800 + 10,139
= -156,000 J

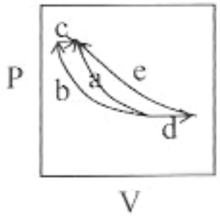
For the Airplane, calculate the KE necessary at 350 km/h
$W_S = \int d(\ m(v^2/2))$
$= m(v^f)^2/2 = 30000\text{kg}*(\ 350000\text{m-hr}/3600\text{hr-s})^2/2$
= 141,782 kJ

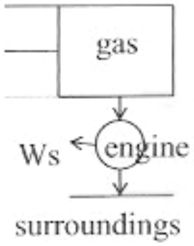
156,000 > 142,000 Therefore the catapult can generate enough work to launch the plane.
Or, one could calculate the final plane velocity for 156,000 kJ.
Sqrt((1.56E8 J)*2/30E3kg) = 102 m/s = 367 km/hr, plenty of speed.

(4.10) We have considered heat and work...

Part (a)







```
P1 = 0.1;
P2 = 0.5;
T1 = 298;
R = 8.314;
Cp = 7/2*R;
T2 = T1*(P2/P1)^(R/Cp);
Tsurr = 273;
% Adiabatic Work
Wadiab = (Cp-R)*(T2-T1);
fprintf('Work and heat in J/mol\n');
fprintf('part (a) adiabatic\n');
fprintf('Q = 0\n');
fprintf('T2(K) = %.2f W_EC(gas)= %.2f\n\n',T2, Wadiab);
```

```
Work and heat in J/mol
part (a) adiabatic
Q = 0
T2(K) = 471.98 W_EC(gas)= 3616.14
```

part (b)

```
% ------- section for step b, c ----------
% Now consider the isothermal step b.
% note that 'log' is natural logarithm in matlab.
% In this and following sections, Q will be written for the Carnot
device.
% Therefore the signs on Q will be opposite for the gas or surroundings.

% Now consider isothermal step, note that 'log' is nat logarithm
WecIsoth_b = R*T1*log(P2/P1); % work for gas
% Qb, Qsurr_b are for the Carnot device
Qb = WecIsoth_b; % for engine on gas side
Qsurr_b = -Tsurr/T1 * Qb; % for engine on surr side
WsIsoth_b = -Qb*(1-Tsurr/T1); % work for engine
fprintf('Step (b) istothermal\n')
fprintf('Q(gas) = %.2f, Q(surr) = %.2f\n', -Qb, -Qsurr_b);
fprintf('W_EC(gas)= %.2f, W_s(engine) = %.2f, W_tot= %.2f\n', WecIsoth_b,
WsIsoth_b,WecIsoth_b+WsIsoth_b);

% Now consider the isobaric step c
WecIsob_c = -R*(T2 - T1);
% Qc, Qsurr_c are for the Carnot device
Qc = -Cp*(T2 - T1); % Q = Delta(H) for an isobaric process
Qsurr_c = Tsurr*Cp*log(T2/T1); % Delta(S_gas) = -Delta(S_surr)=-
Qsurr/Tsurr
WsIsob_c = -Qc-Qsurr_c;

fprintf('Step (c) isobaric\n')
fprintf('Q(gas) = %.2f, Q(surr) = %.2f\n', -Qc, -Qsurr_c);
fprintf('W_EC(gas)= %.2f, W_s = %.2f, W_tot= %.2f\n', WecIsob_c,
WsIsob_c,WecIsob_c+WsIsob_c);
```

```
Step (b) istothermal
Q(gas) = -3987.50, Q(surr) = 3652.98
W_EC(gas)= 3987.50, W_s(engine) = -334.52, W_tot= 3652.98
Step (c) isobaric
Q(gas) = 5062.59, Q(surr) = -3652.98
W_EC(gas)= -1446.46, W_s = 1409.62, W_tot= -36.84
```

Sum of step (b) + (c)

```
Wnet_bc = WsIsoth_b + WecIsoth_b + WecIsob_c + WsIsob_c; % gas + engine
Qnet_bc = Qsurr_b + Qsurr_c; % gas + engine
fprintf('Step (b) + (c)\n')
fprintf('Q(gas) = %.2f, W(gas) = %.2f, Ws(engine)= %.2f\n', -Qb-Qc,
WecIsoth_b + WecIsob_c, WsIsoth_b+WsIsob_c);
fprintf('Q(net) = %.2f, W(net) = %.2f\n\n', Qnet_bc, Wnet_bc);
```

```
Step (b) + (c)
Q(gas) = 1075.09, W(gas) = 2541.04, Ws(engine)= 1075.09
Q(net) = 0.00, W(net) = 3616.14
```

part (c)

```
% -------section for step d e ---
% now consider isobaric step d
WecIsob_d = -R*(T2 - T1);
% Qd, Qsurr_d are for the Carnot device
Qd = -Cp*(T2 - T1); % Q = Delta(H) for an isobaric process
Qsurr_d = Tsurr*Cp*log(T2/T1); % Delta(S_gas) = -Delta(S_surr)=-
Qsurr/Tsurr
WsIsob_d = -Qd-Qsurr_d;

fprintf('Step (d) istothermal\n')
fprintf('Q(gas) = %.2f, Q(surr) = %.2f\n', -Qd, -Qsurr_d);
fprintf('W_EC(gas)= %.2f, W_s(engine) = %.2f, W_tot= %.2f\n', WecIsob_d,
WsIsob_d,WecIsob_d+WsIsob_d);

% now consider isothermal step e
WecIsoth_e = R*T2*log(P2/P1);
% Qe, Qsurr_e for the Carnot device
Qe = WecIsoth_e;
Qsurr_e = -Tsurr/T2 * Qe;
WsIsoth_e = -Qe*(1-Tsurr/T2);
fprintf('Step (e) istothermal\n')
fprintf('Q(gas) = %.2f, Q(surr) = %.2f\n', -Qe, -Qsurr_e);
fprintf('W_EC(gas)= %.2f, W_s(engine) = %.2f, W_tot= %.2f\n', WecIsoth_e,
WsIsoth_e,WecIsoth_e+WsIsoth_e);
```

```
Step (d) istothermal
Q(gas) = 5062.59, Q(surr) = -3652.98
W_EC(gas)= -1446.46, W_s(engine) = 1409.62, W_tot= -36.84
Step (e) istothermal
Q(gas) = -6315.48, Q(surr) = 3652.98
W_EC(gas)= 6315.48, W_s(engine) = -2662.50, W_tot= 3652.98
```

Sum of step (d) + (e)

```
Wnet_de = WsIsoth_e + WecIsoth_e + WecIsob_d + WsIsob_d; % gas + engine
Qnet_de = Qsurr_d + Qsurr_e; % gas + engine
fprintf('Step (d) + (e)\n')
```

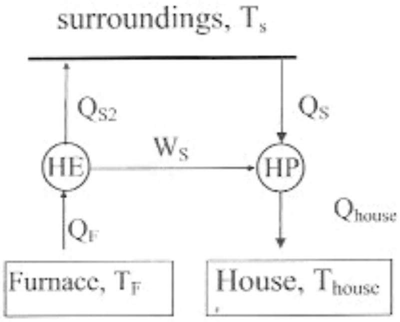
```
fprintf('Q(gas) = %.2f, W(gas) = %.2f, Ws(engine) = %.2f\n', -Qc-Qe,
WecIsoth_e + WecIsob_d, WsIsob_d+WsIsoth_e);
fprintf('Q(net) = %.2f, W(net) = %.2f\n\n', Qnet_de, Wnet_de);
```

```
Step (d) + (e)
Q(gas) = -1252.89, W(gas) = 4869.02, Ws(engine) = -1252.89
Q(net) = 0.00, W(net) = 3616.14
```

Note that all three processes have the same net heat and work interactions with the surroundings. However, the heat and work for the gas are path-dependent.

(4.11) Consider the wintertime heating of a house with a furnace compared to...

a)



Energy Balance over the HE and HP

The signs for Q's and W_S's will be those for the HE and HP, $Q_{house} = -40$ kJ/hr

$W_{S,HE} = -Q_F(1 - T_S/T_F)$ -eq. 1

$W_{S,HP} = Q_S(T_{house}/T_S - 1)$ -eq. 2

Entropy balance over the HP

$Q_S/T_S = -Q_{house}/T_{house}$ -eq. 3

$W_{S,HE} = -W_{S,HP}$ -eq. 4

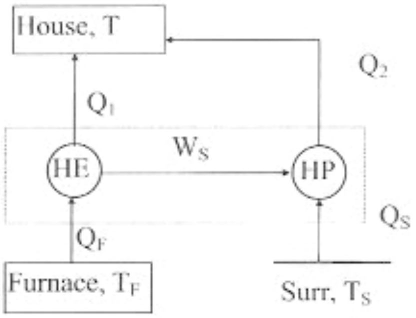
using (4) to relate (1) and (2),

$Q_F(T_F - T_S)/T_F = Q_S(T_{house}-T_S)/T_S$ $Q_F = T_F\, Q_S/T_S * (T_{house}-T_S)/(T_F-T_S)$

insert (3)

$$\text{so}\quad Q_F = -Q_{house}\frac{T_F}{T_{house}}\left(\frac{T_{house}-T_S}{T_F-T_S}\right) = 5.716 \text{ kJ/hr}$$

we can also find $W_{S,HE} = -3.822$ kJ/hr, $Q_S = 36.18$ kJ/hr, $Q_{S2} = -1.893$ kJ/hr b)



Energy Balance about HP and HE

Wnet=0 $\Rightarrow Q_F + Q_S + Q_1 + Q_2 = 0$

Also, $Q_1 + Q_2 = Q_{house} = -40$ kJ/hr

combining, $Q_S = 40 - Q_F$ - eq. 5

Entropy Balance about HP and HE together

$$\frac{Q_F}{T_F}+\frac{Q_S}{T_S}+\frac{Q_1+Q_2}{T_{house}}=0$$

$$\frac{Q_F}{T_F}+\frac{Q_S}{T_S}=\frac{40}{T_{house}} \qquad \text{-eq.6}$$

put eq. 5 and 6,

$$\frac{Q_F}{T_F}+\frac{40-Q_F}{T_S}=\frac{40}{T_{house}} \qquad Q_F\left(\frac{1}{T_F}-\frac{1}{T_S}\right)=40\left(\frac{1}{T_{house}}-\frac{1}{T_S}\right)$$

$$Q_F\left(\frac{T_S-T_F}{T_F T_S}\right)=40\left(\frac{T_S-T}{TT_S}\right)$$

$$Q_F=40\frac{T_F}{T}\left(\frac{T_S-T}{T_S-T_F}\right)=5.716 \text{ kJ/hr}$$

Both the methods require the same heat from the furnace. In fact, and reversible process we conceive with result in the same Q_F.

(4.12) An ideal gas enters a valve at 500 K…

$$\frac{d\underline{S}}{dT}=\dot{m}S^{in}-\dot{m}S^{out}+\frac{d\dot{Q}}{T}+d\dot{\underline{S}}_{gen} \quad \text{where } \frac{d\underline{S}}{dT}=\frac{d\dot{Q}}{T}=0$$

$$d\dot{\underline{S}}_{gen}=\dot{m}(S^{out}-S^{in})$$

as $H^{in}=H^{out}$ and ideal gas, T = same

$$S^{out}-S^{in}=C_P\ln\frac{500}{500}-R\ln\frac{0.5}{3}=14.8967 \text{ J/mol K}$$

$\dot{\underline{S}}_{gen}$ = 0.7448 J/sec K

(4.13) SO2 vapor enters a heat exchanger…

E balance, $m\Delta H = Q$, $\Delta H = Q/m = 700000/45 = 15556$ J/mol
Program Cp constants into ΔH integral. Use trial and error to find T^{out} that gives this ΔH
$T^{out} = 701$ K

isobaric, $S^{out} - S^{in} = \int_{T^{in}}^{T^{out}} \frac{C_P}{T} dT$

program Cp/T integral. Use T from above,
$S^{out} - S^{in} = 29.62$ J/molK
$n(S^{out} - S^{in}) = 45*29.62 = 1333$ J/K/h = $\underline{1.333 \text{ kJ/K/h}}$

(4.14) An ideal gas stream (stream A), Cp = 5/2 R, 50 mole/h, …

E-bal: Σ H n = 0 = 50*2.5*8.314(100-20) + 45*3.5*8.314(t-180)
S-bal: Σ S n = 0 = $50(S_{out} - S_{in})_A + 45(S_{out} - S_{in})_B$
From E-bal: t = 116.5. This is true whether countercurrent or cocurrent
(a) $(S_{out} - S_{in})_A = 2.5*8.314*\ln(373/293) = 5.01762$ J/mole-K
$(S_{out} - S_{in})_B = 3.5*8.314*\ln(389.5/453) = -4.39477$
ΔStot = Sgen = 50(5.01762) - 45(4.39477) = **53.116 J/K**

(b) Since the temperature change is the same for each stream, the results for entropy changes will be identical.

(c) The procedure you follow for throwing away potentially useful energy does not matter. The countercurrent exchanger would be more compact, but same ΔS when the T's are the same. In actual operation the countercurrent can be run with a smaller overall temperature difference than the comparison here, and can generate less entropy.

(4.15) An inventor has applied for a patent on a device …

Basis: 1min. Strategy: ΔH generates xs Q. That Q could make W^{rev}. Enough?

Ebal: $\dot{n}\Delta H = \underline{\dot{Q}} + \underline{\dot{W}} = 1\text{mole/min}*3.5*8.314*(350-500) = \underline{\dot{Q}} - 2000$ J/min

Ebal $\Rightarrow \underline{\dot{Q}} = -4365+2000 = -2365$ J/min

Sbal: $\dot{n}\,\Delta S = \underline{\dot{Q}}/T + \underline{\dot{S}}_{gen} \Rightarrow = \underline{\dot{S}}_{gen} = \dot{n}\,\Delta S - \underline{\dot{Q}}/T$

Sbal $\Rightarrow \underline{\dot{S}}_{gen} = \dot{n}\,\{Cp \ln(T_2/T_1)-R*\ln(P_2/P_1)\} - (-2365/300)$
$= 1*\{3.5*8.314 \ln(350/500)-8.314*\ln(1/2)\} + 2365/300$

$\underline{\dot{S}}_{gen} = +3.2667$ J/K

$\underline{\dot{S}}_{gen} > 0$ for total process. Process is possible.

(4.16) Two streams of air are mixed in a steady-state process…

a) $H^{out}\dot{m}^{out} = \sum H^{in}\dot{m}^{in}$

$T^{out}(1) = 310(0.833) + 250(0.167) = 300\text{ K}$

b) $\frac{d\underline{S}}{dt}=\sum S^{in}\dot{n}^{in}-\sum S^{out}\dot{n}^{out}+\cancel{\frac{\dot{Q}}{T}}+\dot{\underline{S}}_{gen}$

$$\text{Reference state: } S_R=0 \text{ at } 300 \text{ K, } 1 \text{ bar}$$

$$S_A=C_P\ln\frac{T_A}{T_R}=29.3\ln\frac{250}{300}=-5.3420$$

$$S_B=29.3\ln\frac{310}{300}=0.9607$$

$$\dot{S}_{gen}=0-0.167(-5.3420)-0.9607(0.833)=$$

$$=0+0.8921-0.8002=0.0919 \text{ J/s-K}$$

c) By energy balance $Q=-W_S$

By entropy balance, $\frac{Q}{T}=$ (the original $\dot{\underline{S}}_{gen}$), since $\dot{S}_{gen}=0$

$$Q=(295)(0.0919)=27.1 \text{ J/s}$$

$$W_S=-27.1 \text{ J/s}$$

(4.17) Air is flowing at steady-state through a 5 cm diameter pipe at a flowrate....

First, we can evaluate the current process. For a throttle valve, the E-balance shows that the process is isenthalpic. Because it is an ideal gas (H depends only on T), the isenthalpic process is also isothermal.

$$\overset{0}{\cancel{\frac{dnS}{dt}}}=S^{in}\frac{dn^{in}}{dt}-S^{out}\frac{dn^{out}}{dt}+\overset{0}{\cancel{\frac{\dot{Q}}{T}}}+\dot{\underline{S}}_{gen}$$

$$\dot{n}\left(S^{out}-S^{in}\right)=\frac{d\underline{S}_{gen}}{dt}$$

$$S^{out}-S^{in}=C_P\ln\frac{T^{out}}{T^{in}}-R\ln\frac{P^{out}}{P^{in}}$$

$$=29.1\ln\cancel{\frac{500}{500}}-8.314\ln\frac{1}{5}$$

$$=13.3808 \text{ J/mol-K}$$

$$\frac{d\underline{S}_{gen}}{dt}=13.3808(0.35)=4.6833 \text{ J/K min}$$

This is the current entropy generation. We want to replace the current process.

$$\dot{n}\Delta H=\dot{\underline{Q}}+\dot{\underline{W}}_S$$

The state changes are supposed to be the same, this means that ΔH will still be zero. If the state changes are the same, then if we can calculate the heat transfer, we can find the work. The energy balance does not tell us the two unknowns. The entropy balance can be used to find the reversible heat transfer. Draw a boundary around the pipe PLUS the heat transfer device.

$$\overset{0}{\cancel{d(nS)}} = S^{in}dn^{in} - S^{out}dn^{out} + \frac{d\underline{Q}}{T} + \overset{0}{\cancel{d\underline{S}_{gen}}}$$

$$\frac{d\underline{Q}}{dt} = T\left(S^{out} - S^{in}\right)\dot{n} = 298(4.6833) = 1395 \text{ J/min}$$

Note that the heat transfer device will match the surroundings temperature to avoid entropy generation at the boundary.

$$\therefore \frac{d\underline{W}_S}{dt} = -1395 \text{ J/min}$$

pipe diameter doesn't matter.

The net heat transfer from the surroundings is to be converted to work. (This does not mean that the heat transfer to the gas is zero, only that the NET heat transfer to the gas is zero).

(4.18) A common problem in the design of chemical processes is the steady-state compression of gases ...

Solution

E-bal (on 1st compressor): $\Delta H = Q + W = W$

S-bal (on 1st compressor): $\Delta S = 0$

S-bal gives adiabatic reversible ideal gas ⇒ $T^*/T_1 = (P^*/P1)^{R/Cp}$

E-bal gives $W_1 = Cp(T^* - T_1) = Cp\, T_1\, [(P^*/P_1)^{R/Cp} - 1]$

Analogous treatment of 2nd compressor and combination gives:

$W = W_1 + W_2 = Cp\, T_1 \{[(P^*/P_1)^{R/Cp} - 1] + [(P_2/P^*)^{R/Cp} - 1]\}$

To minimize this function, take the derivative and set to zero

$$\frac{dW}{dP} = CpT_1\left[\frac{R/Cp}{P^*}\left(\frac{P^*}{P_1}\right)^{R/Cp} - \frac{R/Cp}{P^*}\left(\frac{P_2}{P^*}\right)^{R/Cp}\right] = 0$$

$(P^*)^2 = P_1 * P_2 \Rightarrow P^* = (P_1 * P_2)^{1/2}$

Can you guess what it would be for a three stage compressor?

(4.19)Steam flowing at steady-state enters a turbine at 400C and 7 MPa.

Assume isentropic

$S'_{out} = S_{in} = 6.4502$ kJ/kg K

$P_{out} = 0.275$ Mpa

From sat P table, interpolating, $S^L = 1.6363$ kJ/kg K

$$S^V = 7.0254 \text{ kJ/kg K}$$

$$q'_{out} = \frac{S - S^L}{S^V - S^L} = 0.893$$

$$H'_{out} = H^L_{out} + q'_{out}(\Delta H^{vap}_{out}) = 547.2 + 0.893(2173.0) = 2487.7 \text{ kJ/kg}$$

$$-W'_S = H_{in} - H'_{out} = 3159.2 - H'_{out} = 671.5 \text{ kJ/kg}$$

$$W_S^{act} = \eta \times W'_S = -570.8 \text{ kJ/kg}$$

$$H_{out} = H_{in} + W_S^{act} = 2588.4 \text{ kJ/kg}$$

$$q^{act}_{out} = \frac{H_{out} - H^L}{H^V - H^L} = 0.94 \text{ at } P_{out}$$

(4.20) A steam turbine operates from 4 MPa to 0.01MPa with 85% eff turbine...

Because the outlet state is known, we must work backwards. We seek the state that results in H^{out} = 2584 kJ/kg and 85% efficiency, ⇒Sin < Sout.
At the outlet: S^{satL} = 0.6492, S^{satV}=8.1488, H^{satL}=191.81, H^{satV} = 2583.86.
q'=(Sin-0.6492)/(8.1488-0.6492); Wact = 2583.86-Hin;

Inlet T	800	700	600	650
H	4142.3	3906.3	3674.9	3790.1
S	7.8523	7.6214	7.3705	7.4988
Outlet				
q'	0.960465	0.929676	0.896221	0.913329
H'	2489.29	2415.64	2335.62	2376.54
ΔH'	-1653.01	-1490.66	-1339.28	-1413.56
Wact	-1405.06	-1322.44	-1091.04	-1206.24
H^{out}	2737	2639	2537	2589

An inlet T of 650°C results in the desired state. At 4 MPa, the saturation temperature is 250, so the superheat is 400°C. The work output is, -1206 kJ/kg.

An inlet T of 650 results in the desired state. The important equations were programmed into Excel to make the trial and error go faster. If the solution didn't fall on a temperature where the steam tables had values, interpolation could be performed, but in this case it wasn't necessary.

(4.21) Steam is fed into an adiabatic turbine at 4 MPa and 500 C...

a) inlet Hin = 3446, Sin = 7.0922, outlet is two phase,
   q' = (7.0922-1.303)/(7.359 – 1.303) = 0.956
   H' = 417.5 + 0.956(2675-417.5) = 2575 kJ/kg
   W_S = ΔH' = 2575 – 3446 = -871 kJ/kg
b) ΔH = η ΔH' = 0.8(-871) = -696 kJ/kg

(4.22)(a) Methane is compressed in a steady-state adiabatic compressor...

Methane compression
$\dot{m}$ =280 kg/hr = 17.5 kmol/hr

Assume isentropic

$S_{in} = S'_{out}$

S_{in} (at 0.1013 MPa, -240 °F) = 2.27 BTU/lb°F

H_{in} =810 kJ/kg

S'_{out} =2.27 BTU/lb°F

P_{out} =0.4 MPa (given)

$\therefore$from chart, H'_{out} =920 kJ/kg

$W^{rev} = H'_{out} - H_{in}$ =110 kJ/kg

$W^{act} = \dfrac{W^{rev}}{\eta}$ = (110 / 0.87) = 126 kJ/kg

Work required per mole of Methane = 126×(16/1000) = 2.016 kJ/mol

$H^{act}_{out} = H_{in} + W^{act}$ =936 kJ/kg

at H = 936 kJ/kg and P = 0.4 MPa, S^{act}_{out} = 2.28 kJ/kg K

Entropy Balance

$0 = \underline{S}_{in} - \underline{S}_{out} + \dot{\underline{S}}_{gen}$

$S^{out} - S^{in}$ = (0.01 BTU/lb °F)(2.205 lb/kg)(kJ/0.94805 BTU)(1.8 °F/°C)
= 4.186E-2 kJ/kg °C

$\dot{\underline{S}}_{gen} = \dot{m}(4.186E\text{-}2)$ = 11.72 kJ/hr-K

Work in kW = 126 ×280 (kg/hr)×(1/3600) = 9.8 kW

(b) Methane is compressed..

200K = -100 °F

H'_{out} = 1155 H'_{in} = 980

$\Delta H'$ = 175 $\Delta H = \dfrac{175}{0.87}$ = 201

H^{out} = 980 + 201 = 1181

$W = \Delta H$ = 201 kJ/kg

MW of CH_4 = 16

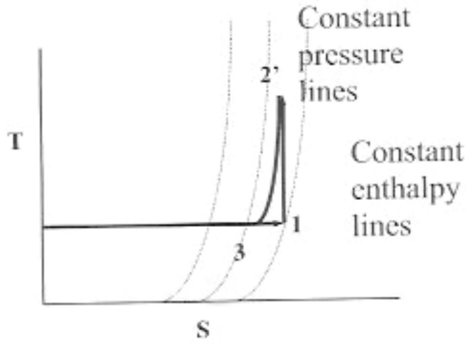
$\dot{\underline{W}}$ = 201 (kJ/kg)(16 kg/kmol)(17.5 kmol/h)(h/3600 s) = 15.6 kW

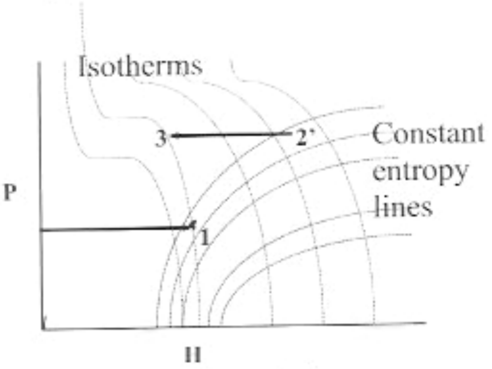
$\dot{S}_{gen} = \left(S^{out} - S^{in}\right)\dot{n}$ = (2.58 – 2.565)(4.186)(280) = 17.6 kJ/kh

(4.23) Methane is to be compressed from 0.05 MPa and –120F…

This problem requires the use of the T-S chart and or the P-H chart given in the appendix for methane gas.

A schematic sketch of both the charts and usage is shown below.





Assumption: Isentropic path for compressor, isobaric path for the inter stage cooler

From the chart,

$H_1 = 960$ KJ/kg,

$S_1 \approx 2.62$ Btu/lb$_m$

$S_2 \approx 2.62$ Btu/lb$_m$

$T_2 \approx 260\ ^{\circ}$F

$H_2 \approx 1440$ KJ/kg

$H_3 \approx 940$ KJ/kg

A) $\underline{W}_S = H_2 - H_1 = 1440 - 960 = 480$ KJ/kg

B) The temperature at the exit of the first compressor is around 260°F

C) The amount of cooling required can be calculated using isobaric change in enthalpy $\underline{Q} = 1440 - 940 = 500$ KJ/kg

(4.24) A steady stream (1000 kg/hr) of air flows

$0 = m_{air}(H^{in} - H^{out})_{air} + m_{water}(H^{in} - H^{out})_{water} + \underline{W}_S.$

$0 = (1000\text{ kg/hr})(29\text{ kJ/kmol }^{\circ}\text{C})(300 - 425\ ^{\circ}\text{C})(\text{kmol}/28.84\text{kg})$
$+ 1500\text{ kg/hr}(4.184\text{ kJ/kg }^{\circ}\text{C})(-20) + \underline{W}_S \Rightarrow \underline{W}_S = 251{,}213\text{ kJ/hr} = 69\text{ kW}$

For the minimum work, assume that the surroundings are at 300 K (other assumed temperatures will change the answer slightly.

$0 = m_{air}(H^{in} - H^{out})_{air} + \underline{Q} + \underline{W}_S$

S-bal

$0 = m_{air}(S^{in} - S^{out})_{air} + \underline{Q}/T \Rightarrow \underline{Q} = T\, m_{air}(S^{out} - S^{in})_{air}$

$\underline{Q} = (300)(1000)\{29\ln(425/300) - 8.314\ln(1/0.1)\}(\text{kmol}/28.84\text{kg}) = -94065\text{ kJ/hr}$

$\underline{W}_S = (1000\text{ kg/hr})(29\text{ kJ/kmol }^{\circ}\text{C})(425 - 300\ ^{\circ}\text{C})(\text{kmol}/28.84\text{kg}) + 94065 = 219{,}758\text{ kJ/hr} = 61$ kW

(4.25) Propane is to be compressed from 0.4 MPa and 360 K …

Key
1 - inlet to first compressor
2 - between compressor I and cooler
3 - between compressor II and cooler
4 - after compressor II

a) At state 1, T_1 =360 K (188°F), P_1 =0.4 MPa
$\therefore H_1 = 1060$ kJ/kg, $S_1 = 1.48$ Btu/lb-F = 6.18 kJ/kg K

For compressor I
Isentropic calculation: $S_2' = S_1 = 1.48$ Btu/lb-F = 6.18 kJ/kg K
$P_2 = 1.2$ MPa
$\therefore H_2' = 1138$ kJ/kg

Irreversibility:
$\eta = \dfrac{H_2' - H_1}{H_2 - H_1} = 0.8 \quad \Rightarrow H_2 = 1158$ kJ/kg
At H_2 = 1158 kJ/kg and $P_2 = 1.2$ Mpa,
T_2 = 280°F = 411 K

$W_I = H_2 - H_1$ = 1158-1060 = 98 kJ/kg propane

b) T_2 = 280°F = 411 K as noted above

c) At state 3, $P_3 = 1.2$ MPa, $T_3 = 360$ K, and $H_3 = 1045$ kJ/kg
$|Q_{cooler}| = |H_2 - H_3|$ = 113 kJ/kg propane

(4.26) (a) A steam turbine in a small electric power plant is designed …

SOLUTION:

a) $\dfrac{d\underline{S}}{dt} = \Sigma m_k\, S_k + \dfrac{\dot{\underline{Q}}}{T} + \dot{\underline{S}}_{gen}$ Steady state, $\dfrac{d\underline{S}}{dt} = 0$; Adiabatic, $\dfrac{\dot{\underline{Q}}}{T} = 0$; And $\dot{\underline{S}}_{gen} = 0$

$\sum \dot{m}_k S_k = \dot{m}(S_2 - S_1) = 0 \Rightarrow S_2 = S_1$

S_1(6MPa,500°C) = 6.8826 kJ/kg-K; H_1=3423.1 kJ/kg

T_2(0.1 MPa,S_2) = 99.61 °C; $q = \dfrac{6.8826 - 1.3028}{7.3584 - 1.3028} = 0.921$,

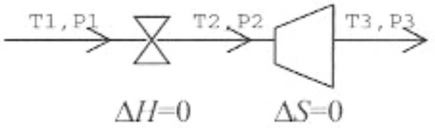
H_2' = 417.50 + 0.921*(2257.45) = 2497.58 kJ/kg

$\underline{\dot{W}} = \dot{m}\Delta H$ =5000 kg/h*(2497.58 – 3423.1) kJ/kg
= -4,627,610.7 kJ/h = -1,285 kW

b) $\underline{\dot{W}}_{\text{act}} = 80\%\underline{\dot{W}} = \dot{m}\Delta H \Rightarrow H_2 = 0.8^*\underline{\dot{W}}/\dot{m} + H_1$
H_2 = 0.8(-4627610.7) / 5000 + 3423.1 = 2682.7 kJ/kg, Actual state is superheated.

$$T_2 = 100 + \left(\frac{2682.7 - 2675.8}{2776.6 - 2675.8}\right)(150 - 100) = 103.4\ ^{\circ}\text{C}$$

S_2= 7.3783 kJ/kg-K (after interpolation)
S_{gen} = 5000 * (7.3783 – 6.8826) = 2478.3 kJ / kg-K



c) $H_2 = H_1$(6MPa,500°C) = 3423.1 kJ/kg
From Superheated Steam tables, and interpolate to get T_2 (3 MPa, H_2) = 484.8 K and S_2= 7.1903 kJ/kg-K, $S_2 = S_3$ = 7.1903 kJ/kg-K
You need to recalculate quality at the outlet of P=1 bar
q_3 = (7.1903 – 1.3026) / (7.3589 – 1.3026) = 0.972
H_3= 417.5 + 0.972*(2257.45) = 2612.1 kJ/kg
$\underline{\dot{W}} = \dot{m}\,\Delta H_{2\rightarrow 3}$ = 5000(2612.1 – 3423.1) = -4,055,000 kJ/hr = -1126.4 kW

$$\frac{d\underline{S}}{dt} = \Sigma \dot{m}_k\, S_k + \frac{\dot{Q}}{T} + \dot{\underline{S}}_{gen}$$

$$\Rightarrow \dot{\underline{S}}_{gen} = \dot{m}\Delta S = 5000 \text{ kg/h } (7.1903 - 6.8826) = 1538.5 \text{ kJ/h-K}$$

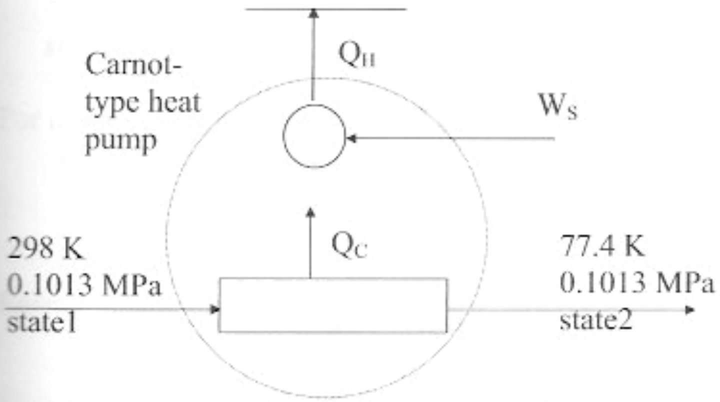
(4.27) Steam is used in the following turbine system to generate electricity…

shaded numbers known at start.

state	T(C)	P(MPa)	H	S	q
1	550	8	3521.8	6.8799	
2'		5	3361.3	6.8799	
2		5	3393	6.9228	
3'		0.8	2932	6.9228	
3		0.8	3024		

a) State 2 between 450 and 500 C,
H_2'= 3317.2 + (6.8799-6.821)/(6.9781-6.821)*(3434.7-3317.2) = 3361.3 kJ/kg
ΔH' = -160.547, W_S = ΔH = 0.8(-160.5) = -128 kJ/kg,, H_2 = 3393 kJ/kg
S_2 = 6.821 + (3393 – 3317.2)/(3434.7-3317.2)*(6.9781-6.821) = 6.9228 kJ/kgK
$\underline{W_S}$ = 300*(-128) = $\underline{-38400 \text{ kJ/h}}$
b) now step across next turbine, outlet is superheated since S > S^{satV} at 0.8 MPa, will be between 200 and 250C.
H_3'= 2839.7 + (6.9228-6.1876)/(7.0401-6.1876)*(2950.4-2839.7) = 2932 kJ/kg
ΔH' = 2932-3393 = -461, W_S = ΔH = 0.8(-461) = -369 kJ/kg, H_3 = 3024 kJ/kg
$\underline{W_S}$ = 300(0.85)*(-369) = $\underline{-94095 \text{ kJ/h}}$
c) E-bal m(H^{out} – H^{in}) = $\underline{Q}$
$H^{in} = H^{sat} + V\Delta P$ = 720.8 + 1(8-0.8) = 727.9 kJ/kg
$\underline{Q}$ = 300(3521.8 – 728) = 837,900 kJ/h

(4.28) Liquid nitrogen is useful for medical purposes and for research …



Without property tables or chart, we can calculate H and S assuming ideal gas law is valid.

a) Steady-state flow system

Energy balance

$$0 = H^{in}\dot{n}^{in} - H^{out}\dot{n}^{out} + \underline{\dot{Q}}_H + \underline{\dot{W}}_S = -\Delta H\dot{n} + \underline{\dot{Q}}_H + \underline{\dot{W}}_S$$

Entropy balance

$$0 = S^{in}\dot{n}^{in} - S^{out}\dot{n}^{out} + \frac{\underline{\dot{Q}}_H}{T_{H,pump}} + \underline{\dot{S}}_{gen} = -T_{H,pump}\Delta S\dot{n} + \underline{\dot{Q}}_H \qquad \underline{\dot{S}}_{gen} = 0 \text{ for reversible}$$

Through out $\underline{\dot{S}}_{gen} = 0$ because the minimum work will be for a reversible process.

Strategy – solve entropy balance for $\underline{\dot{Q}}_H$, then put in energy balance to find $\underline{\dot{W}}_S$.

We need ΔH, ΔS

$$\Delta H = \int_{298}^{77.4} C_P dT - \Delta H^{vap}$$

assuming $C_P = \frac{7}{2}R$

$$\Delta H = \frac{7}{2}(8.314)(77.4-298) - 5577 = -6419 - 5577 = -11996 \text{ J/mol}$$

$$\Delta S = \int_{298}^{77.4} \frac{C_P}{T} dT - \frac{\Delta H^{vap}}{T^{vap}}$$

$$= \frac{7}{2}(8.314)\ln\frac{77.4}{298} - \frac{5577}{77.4} = -111.283 \text{ J/mol K}$$

$$\underline{\dot{Q}}_H = T_{H,pump}\Delta S\dot{n} \qquad \text{Set } T_{H,pump} = 298 \text{ K} = T_{surr} \text{ for minimum work}$$

$$= 298(-111.283)(1)$$

$$= -33162 \text{ J/min}$$

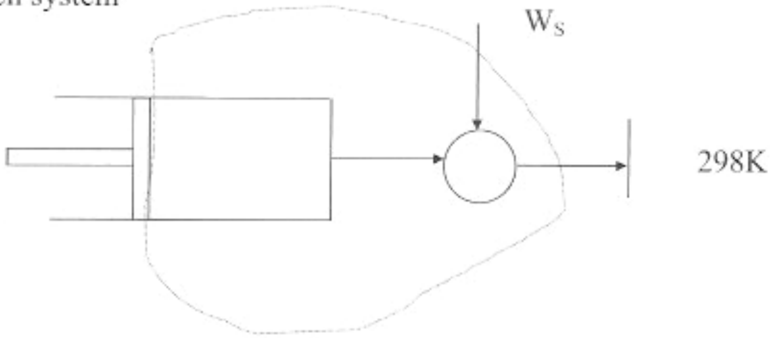
Energy balance

$\underline{W}_S = \Delta H\dot{n} - \underline{\dot{Q}}_H$ = -11996(1) + 33162

=21.17 (kJ/min)(min/60s)

=0.353kW

b) Batch system



Constant pressure process, closed system

Energy balance

$dU = dQ_H + dW_S + dW_{EC} = dQ_H + dW_S - PdV$

Rearranging $dU + PdV + VdP = dH$

So for a constant pressure process,

$dH = dQ_H + dW_S$

$\Delta H = Q_H + W_S$

Entropy balance

$dS = \dfrac{dQ_H}{T_{H,pump}}$

Balances will be numerically the same.

(4.29) Propane flows into a steady-state process at 0.2 MPa and 280 K.

Refer to the P-H diagram for propane

Inlet at Tin = 280 K = 44.33 °F, Pin = 0.2 MPa in the vapor phase region

Sin = 1.40 (BTU/lb-°F) (1kJ/0.94805 BTU) (1lb/0.4536 kg) (1.8F/K) =
1.4(4.186) = 5.86 kJ/kg-K

Hin = 920 kJ/kg

Outlet at Tout = 300 K = 80.33 °F

Sat'd liquid at this temperature gives Pout = 0.98 MPa

Sout = 1.075 BTU/lb-°F * 4.186 = 4.50 kJ/kg-K
Hout = 590 kJ/kg

S-balance,

$$\cancel{d\underline{S}/dt} = m(\underline{S}_{in} - \underline{S}_{out}) + d\underline{Q}/T_{surr} + \cancel{\underline{S}_{gen}}$$

steady state　　　　　　　　　　　　reversible

For minimum work, use T = 295 K to match T_{surr},

$\underline{Q}/T_{surr} = m(\underline{S}_{out} - \underline{S}_{in})$

$\underline{Q}$ = 1000kg/hr * 298 K * (4.50 – 5.86) kJ/kg-K * (1 hr/ 3600sec)
= -111.4 kJ/sec = -111.4 kW

Energy balance,

$$0 = m_{in}H_{in} - m_{out}H_{out} + \underline{Q} + \underline{W}_S$$

$$m_{in} = m_{out}$$

$$\underline{\mathbf{W}}_S = m(H_{out} - H_{in}) - \underline{Q}$$

= 1000kg/hr * (590 –920) kJ/kg * (1hr / 3600 sec) - (-111.4 kW) = **19.7** kW

(4.30) Propane (1000kg/hr) is to be liquefied following ....

(a) Inlets: T_1 = 80 °F ; T_2=116°F
For compressor 1: W = (1070-958)/0.8 = 140 ⇒ Hout = 958+140 = 1098
For compressor 2: W = (1080-980)/0.8 = 125 ⇒ Hout = 980+125 = 1105

(b) 140/0.8*1000kg/hr = 175,000 kJ/hr*1hr/3600s *(1hp/0.7457kW) = 65hp
125 *65/140 = 58hp

(c) Q1 = 980 - 1098 = -118 kJ/kg; Q2 = 650(116°F, 4.5MPa) -1105 = -455 kJ/kg

(d) q = (650-440)/(850-440) = 52%, -42°F. ⇒ 1-q=48%

FYI: Solving by IG EOS for compressor work:
W_I=8.851*8.314*300*((0.65/0.1)^(1/8.851)-1)/0.8/44= 147.7 => Hout=1106
W_{II}=8.851*8.314*300*((4.5/0.65)^(1/8.851)-1)/0.8/44= 153.2 => Hout=1133

The result for W_I is closer than W_{II} because the pressure is lower, hence more like an ideal gas. From this example, we learn that the IG EOS has significant limitations, especially at higher pressures. We also learn that reading the chart has a certain amount of variability. In conclusion, we would prefer a method that has the precision of the IG EOS and the accuracy of the chart. The PR EOS of Unit II is such a method.

(4.31) A heat exchanger operates with the following streams...

a) We must provide heat exchange such that

$0 = -m_{water}\Delta S_{water} - m_{org}\Delta S_{org} + \underline{Q}/298 = 0$

all streams are isobaric,

$\underline{Q}$ = 298*(30*4.184ln(343.2/293.2)+41.8*2.5ln(313.2/373.2)) = 432 kJ/h

E bal, 0 = $-m_{water}\Delta H_{water} - m_{org}\Delta H_{org} + \underline{Q} + \underline{W}$
the enthalpy terms already balance, therefore, $\underline{W} = -\underline{Q}$ = -432 kJ/h

b) $0 = -m_{water}\Delta S_{water} - m_{org}\Delta S_{org}$
30*4.184ln(T_{water}/293.2) = -41.8*2.5ln(313.2/373.2)
T_{water} = 339.3K

E bal, 0 = $-m_{water}\Delta H_{water} - m_{org}\Delta H_{org} + \underline{W}$

30(4.184)(339.3 – 293.2) + 41.8(2.5)(313.2 – 373.2) = $\underline{W}$ = <u>-484 kJ/h</u>

(4.32) Presently, benzene vapors are condensed in a heat exchanger using...

Properties of benzene are available in the appendix. Let us use average heat capacities. For liquid water, Cp = 75.4 J/molK. Recognizing the boiling point of benzene is 80.1C, calculate the heat capacity in the middle of the needed range: for liquid benzene at 70C, Cp = 18.13*8.314 = 150.7 J/molK, for vapor benzene at 100C, Cp = 104.6 J/molK

a) constant pressure process, ΔH_{water} = Cp(40-10) = 75.4(30) = 2262 J/mol
for $\Delta H_{benz} = Cp^V(80.1 - 120) - \Delta H^{vap} + Cp^L(50-80.1)$ =
104.6(80.1-120) – 30765 + 150.7(50-80.1) = -39475 J/mol
E bal
$m\Delta H_{water} + m\Delta H_{benz} = 0 = m(2262) + 100{,}000(-39475)$,
m = (1.745E6 mol/h)*(0.018 kg/mol) = 3.14 E4 kg/h

b) If reversible, the entropy balance requires
$m\Delta S_{water} + m\Delta S_{benz} = 0$

ΔS_{water} = 75.4ln(313.2/283.15) = 7.6052 J/molK

$\Delta S_{benz} = Cp^V\ln(353.3/393.2) - \Delta H^{vap}/353.3 + Cp^L\ln(323.2/353.3)$ =
104.6(-0.107) – 30765/353.3 +150.7(-0.08946) = -111.75 J/molK

S balance
m7.6052 + 100,000(-111.75) = 0, m = 1.47E6 mol/h = 2.65E4 kg/h

reduction in water flow, (2.64-3.14)/3.14 = -0.16, 16% reduction if reversible.

(4.33) A Hilsch vortex tube is an unusual device that takes an inlet gas stream and produces a hot stream and a cold stream with out moving parts. ...

a)

$$\dot{n}_A H_A - \dot{n}_B H_B - \dot{n}_C H_C = 0$$

i.g. Ref. State T=310 K, P = 5 bar

$$H_A = 0$$

$$H_B = 29.1(260-310) = -1455$$
$$H_C = 29.1(315-310) = 145.5$$
$$\frac{\dot{n}_B}{\dot{n}_C} = \frac{145.5}{1455} = 0.1$$
$$\dot{n}_B + \dot{n}_C = 3.2 = \dot{n}_A \text{ mol/min}$$

Combining the last two eqns to eliminate $\dot{n}_B$, $1.1\dot{n}_C = 3.2$

$$\dot{n}_C = 2.909 \text{ mol/min}$$
$$\dot{n}_B = 0.2909 \text{ mol/min}$$

b) Energy Balance

$$0 = \dot{n}_A S_A - \dot{n}_B S_B - \dot{n}_C S_C + \dot{S}_{gen}$$

Ref. State is condition A

$$S_B = C_P \ln\frac{T_B}{T_A} - R\ln\frac{P_B}{P_A} = 29.1\ln\frac{260}{310} - 8.314\ln\frac{1}{5} = 8.2624 \text{ J/mol K}$$
$$S_C = C_P \ln\frac{T_C}{T_A} - R\ln\frac{P_C}{P_A} = 29.1\ln\frac{315}{310} - 8.314\ln\frac{1}{5} = 13.846 \text{ J/mol K}$$
$$S_A = 0 \text{ (at ref. state)}$$
$$\dot{S}_{gen} = 0.2909(8.2624) + 2.909(13.846) - 3.2(0) = 42.68 \text{ J/mol K}$$

c) This is a reversible heat exchanger

$$0 = \dot{n}_B S_B + \dot{n}_C S_C - \dot{n}_{B'} S_{B'} - \dot{n}_{C'} S_{C'}$$
$$\dot{n}_B \Delta S_B + \dot{n}_C \Delta S_C = 0$$

neglect pressure drop through HX

$$\dot{n}_B C_P \ln\frac{T_D}{T_B} + \dot{n}_C C_P \ln\frac{T_D}{T_C} = 0$$
$$(\dot{n}_B + \dot{n}_C)\ln T_D - \dot{n}_B \ln T_B - \dot{n}_C \ln T_C = 0$$
$$3.2\ln T_D = 0.2909\ln 260 - 2.909\ln 315 = 0$$
$$\ln T_D = 5.7349$$
$$T_D = 309.5$$

d) E-balance

$$0 = -\dot{n}_B \Delta H_B - \dot{n}_C \Delta H_C + \underline{\dot{W}}_S$$
$$\Delta H_B = 29.1(309.5-260) = 1440.5$$
$$\Delta H_C = 29.1(309.5-315) = -160.05$$
$$\underline{\dot{W}}_S = 0.2909(1440.5) + 2.909(-160.05) = -46.54 \text{ J/min}$$
$$\underline{\dot{W}}_S = -46.54 \text{ (J/min)*(min/60 sec)} = -0.776 \text{ W}$$

$\dot{S}_{gen}$ doesn't change

c) If mix directly

$$\dot{n}_B \Delta H_B = \dot{n}_C \Delta H_C$$

Cancelling C_P

$$(0.2909)(T_D - 260) = (2.909)(T_D - 315)$$

$$(2.909 - 0.2909)T_D = 2.909(315) - 0.2909(260)$$

$$T_D = 310 \text{ K} \quad \text{close to answer from above}$$

$\Delta \dot{S}_{gen}$ will be slightly larger

$$(\dot{S}_{gen})_{HX} = \dot{n}_B \Delta S_B + \dot{n}_C \Delta S_C$$

$$= 0.2909 C_P \ln\frac{310}{260} + 2.909 C_P \ln\frac{310}{315}$$

$$= C_P(0.051167 + (-4.6545E-2))$$

$$= 0.1345 \text{ J/K min}$$

$$\boxed{(\dot{S}_{gen})_{tot} = 42.68 + 0.1345 = 42.81 \text{ J/K min}}$$

(4.34) Methane gas is contained in a 0.65 m^3 gas cylinder at 6.9 MPa…

a) molar entropy of gas will stay constant during depressurization,

$\Delta S = C_p \ln(T_2/T_1) - R \ln(P_2/P_1) = 0$, $T_2 = T_1(P_2/P_1)^{R/Cp} = 300(0.5/6.9)^{1/4.298} = 163$ K

$n = P\underline{V}/RT = 0.5(650{,}000)/8.314/163 = 239$ mols

(b)300 K = 80 °F

$S^f = S^i = 2.22$ BTU/lb °F , follow isentrope

$T^f = -190$ °F = 150 K

reading density lines, $\rho^f = 0.41$ lb/ft^3 (kg/2.20462lb)(35.315ft^3/m^3) = 6.57 kg/m^3

$m^f = \rho^f \underline{V} = (6.57)0.65 = 4.27$ (kg)(kmol/16 kg) = 0.27 kmol = 270 moles

(4.35) "Fix-a-flat" uses a 500ml can to fill a flat 40,000ml tire. Fill tire 1→3bar.

For Can: $\Delta S = 0$ — Entropy balance for leaky tank

For Overall: $n_i^C + n_i^T = n_f^C + n_f^T$; — Mass Balance

$n_i^C U_i^C + n_i^T U_i^T = n_f^C U_f^C + n_f^T U_f^T$ — Energy Balance

Because ideal gas: $n = P\underline{V}/RT$, $U = Cv(T - T_{ref})$; note that mass balance implies that T_{ref} cancels out on both sides of equal sign. So, the energy balance becomes:

$$\frac{P_i^C \underline{V}^C}{RT_i^C} CvT_i^C + \frac{P_i^T \underline{V}^T}{RT_i^T} CvT_i^T = \frac{P_f^C \underline{V}^C}{RT_f^C} CvT_i^C + \frac{P_f^T \underline{V}^T}{RT_f^T} CvT_f^T \Rightarrow P_i^C \underline{V}^C + P_i^T \underline{V}^T = P_f(\underline{V}^C + \underline{V}^T)$$

$$\Rightarrow P_i^C = [3(40500) - 40000]/500 = 162 \text{ bars}$$

For the can, the entropy balance gives:

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$$\frac{T_f^C}{T_i^C}=\left(\frac{P_f^C}{P_i^C}\right)^{2/7}=\left(\frac{3}{162}\right)^{2/7}\Rightarrow T_f^C=96\text{ K}$$

For the tire, the MASS balance gives:

$$\frac{P_i^C\underline{V}^C}{RT_i^C}+\frac{P_i^T\underline{V}^T}{RT_i^T}=\frac{P_f^C\underline{V}^C}{RT_f^C}+\frac{P_f^T\underline{V}^T}{RT_f^T}$$, Solving ⇒ $T_f^T=308$ K.

(4.36) A more interesting problem is "fix-a-flat with a light bulb."

For Can: $\Delta S=0\ \Rightarrow S_i^C=S_f^C$ Entropy balance

For Overall: $n_i^C+n_i^T=n_f^C+n_f^T$; Mass Balance

$n_i^C\,U_i^C+n_i^T\,U_i^T+W=n_f^C\,U_f^C+n_f^T\,U_f^T$ Energy Balance

$n_i^C\,S_i^C+n_i^T\,S_i^T=n_f^C\,S_f^C+n_f^T\,S_f^T$ Entropy Balance

Because ideal gas: $n=P\underline{V}/RT$ so the MASS balance gives:

$$\frac{P_i^C\underline{V}^C}{RT_i^C}+\frac{P_i^T\underline{V}^T}{RT_i^T}=\frac{P_f^C\underline{V}^C}{RT_f^C}+\frac{P_f^T\underline{V}^T}{RT_f^T}$$ **eqn (1)**

$U=Cv(T-T_{ref})$; note that mass balance implies that T_{ref} cancels out on both sides of equal sign. So, the energy balance becomes:

$$n_i^CT_i^C+n_i^TT_i^T+W/Cv=n_i^CT_i^C+n_f^TT_f^T\Rightarrow WR/Cv=P_i^C\underline{V}^C+P_i^T\underline{V}^T-P_f(\underline{V}^C+\underline{V}^T)$$

Rearranging the overall entropy balance and substituting the can entropy balance:

$(n_i^C-n_f^C)S_i^C=\ n_f^T\,S_f^T-n_i^T\,S_i^T$

By mass balance, $(n_f^T-n_i^T)=(n_i^C-n_f^C)$

$\Rightarrow(n_f^T-n_i^T)S_i^C=\ n_f^T\,S_f^T-n_i^T\,S_i^T\Rightarrow n_f^T(S_i^C-S_f^T)=n_i^T(S_i^C-S_i^T)$,

$$n_f^T\left[Cp\ln(T_f^C/T_f^T)-R\ln(P_f^C/P_f^T)\right]=n_f^T\left[Cp\ln(T_f^C/T_f^T)\right]=n_i^T\left[Cp\ln(T_i^C/T_i^T)-R\ln(P_i^C/P_i^T)\right]$$

$$n_f^T/n_i^T=(P_f^T/P_i^T)\,(T_i^T/T_f^T)\ \Rightarrow\ \left(\frac{P_f^T}{P_i^T}\right)\left(\frac{T_i^T}{T_f^T}\right)\left[Cp\ln\left(\frac{T_f^C}{T_f^T}\right)\right]=\left[Cp\ln\left(\frac{T_i^C}{T_i^T}\right)-R\ln\left(\frac{P_i^C}{P_i^T}\right)\right]$$ **eqn (2)**

For the can, the entropy balance gives:

$$\frac{T_f^C}{T_i^C}=\left(\frac{P_f^C}{P_i^C}\right)^{2/7}$$ **eqn (3)**

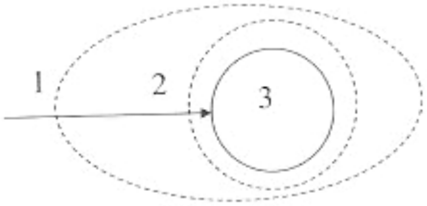
3 eqns and 3 unknowns (P_f, T_f^C, T_f^T)

Pf	1.816	PiC	PiT	TiC	TiT	Cp	Rg	VC	VT
TfC	83.150401	162	1	300	300	7	2	500	40000
TfT	185.11092								
Pf	1.816308								

W = (16.2*500+0.1*(40000)-0.1816*40500) (Cv/R) = 12338 Joules to light a 12W bulb for 1000 seconds.

Note: the Pf is a bit low at 185 K but seal it up and wait to equilibrate with air. Then Pf = 1.816*300/185 = 2.94 bar, almost like with no turbine, but we have stolen energy from the air (and the tire may go through a brittle transition that makes it explode).

(4.37) A 1-m^3 tank is to be filled using N_2 at 300K and 20 MPa



$S_2 = S_1 \quad P_3 = P_2$

it is not clear that $T_3 = T_2$. If so, then the process will be reversible, $S_3 = S_2$. Let us first work the problem as if not recognizing.

S-bal around expander gives $\boxed{T_2 = T_1\,(P_2/P_1)^{R/Cp} = T_1\,(P_3/P_1)^{R/Cp}}$ (1)

Boudary around tank (3), set $T_R = 400$ K, $H_R = 0$;

$d\underline{U}_3 = H_2 dn_3 \rightarrow \boxed{d\,(n_3U_3)\,/\,d\,n_3 = H_2 = Cp\,(T_2 - T_R)}$ (2)

which can be numerically integrated for the lumped variable n_3U_3.

Also $\boxed{P_3 = (n_3T_3)R/\underline{V}}$ (3)

and at all times

$U_R = H_R - (PV)_R = -RT_R$

$n_3U_3 = n_3[Cv(T_3 - T_R) - RT_R] = Cv(n_3T_3) - n_3CpT_R$ (4a)

which rearranges to (keeping n_3T_3 as a lumped variable)

$\boxed{n_3T_3 = (n_3U_3 + n_3CpT_R)\,/\,Cv}$ (4b)

Boundary around entire system

$d\,n_3U_3 = H^{in}\,dn_3 + d\underline{W}_S$ integrates to

$\boxed{\underline{W}_S = n_3U_3 - H^{in}n_3}$ (5)

The spreadsheet below shows that $T_2 = T_3$, so we could have assumed reversibility. However, if there is some mass initially in the tank at a measurable T, then irreversibilities will result (not shown).

Numerical integration using RungeKutta.xls where n_3 is adjusted to give a final pressure of 20 MPa. The final T is 300K, and 20 MJ of work are produced.

Enter Independent Variable Information

Name	*Initial Value*	*Final Value*
n3.	0.0001	8018.6
Num of Steps		10000
Step Size (calc'd)		0.80185999
Number of Points in Summary		20

Enter Constants, Initial Values, Formulas, and Differential Equations

Name	*Input*	*Display Formula*
R	8.314	
Vbar	1.00E+06	
n3T3	0.056000481	=(B16+7*B9*indep*B13/2)/(5*B9/2)
P2	4.66E-07	=B11*B9/B10
TR	400	
T1	300	
T2	1.978135558	=B14*(B12/20)^(2/7)
n3U3	0.00001	
n3Hin	-0.29099	=indep*7*B9/2*(B14-B13)
W	0.291	=B16-B17
d(n3U3)/d(n3.)	-11582.03823	=7*B9/2*(B15-B13)

Summary of Output

n3.	n3U3	n3T3	P2	T2	n3Hin	W
0.0001	0.00001	0.0560005	4.66E-07	1.978136	-0.29099	0.291
400.9301	-3912435	36287.281	0.301692	90.51124	-1166666	-2745768.31
801.8601	-7342849	95765.279	0.796193	119.4312	-2333333	-5009516.71
1202.79	-10488519	168942.81	1.404591	140.4609	-3499999	-6988520.22
1603.72	-13413678	252729.48	2.101193	157.591	-4666665	-8747013.03
2004.65	-16154041	345406.96	2.871713	172.3042	-5833331	-10320710.2
2405.58	-18733065	445846.75	3.70677	185.3398	-6999997	-11733067.7
2806.51	-21167612	553237.57	4.599617	197.1276	-8166664	-13000948.1
3207.44	-23470523	666961.56	5.545118	207.943	-9333330	-14137193.5
3608.37	-25651982	786528.85	6.539201	217.9744	-10499996	-15151985.9
4009.3	-27720304	911539.31	7.578538	227.3571	-11666662	-16053642
4410.23	-29682441	1041658.5	8.660349	236.1923	-12833328	-16849112.6
4811.16	-31544308	1176601.9	9.782268	244.5576	-13999995	-17544313.8
5212.09	-33311017	1316123.5	10.94225	252.5143	-15166661	-18144356
5613.02	-34987036	1460008.3	12.13851	260.1117	-16333327	-18653708.8
6013.95	-36576315	1608066.3	13.36946	267.3901	-17499993	-19076322
6414.88	-38082377	1760128	14.6337	274.3827	-18666659	-19415717.5
6815.81	-39508385	1916041.2	15.92997	281.1178	-19833326	-19675059.4
7216.74	-40857202	2075668.3	17.25711	287.6192	-20999992	-19857210.2
7617.67	-42131432	2238883.8	18.61408	293.9072	-22166658	-19964774.6
8018.6	-43333459	2405236.4	19.99714	299.9877	-23330991	-20000134.9

Had we recognized that reversibility existed,

$S_3^f = S^{in} \rightarrow T_2^f = T_3^f = T_1(20/20)^{R/Cp} = 300$

$n_3 = P_3\underline{V}/RT_3 = 20\ (1E6)\ /\ 8.314/300 = 8018.6$ mol

by (5)

$W_S = (U^f - H^{in})n_3 = (CvT^f - CpT^{in})n_3 = R\ [(5/2)\ T^f - (7/2)\ T^{in}]\ n_3$

$\boxed{W_S = -8.314(300)(8018.6) = -20 \text{ MJ}}$

(4.38) Two well-insulated tanks are attached as shown in the figure below…

Initially

Left hand side, $n_A{}^i = P\underline{V}/RT = 0.5(5E6)/8.314/500 = 601.4$ mol

Right hand side, $n_B{}^i = P\underline{V}/RT = 0.09(7E6)/8.314/300 = 252.6$ mol

Left hand side will be isentropic, the temperature and pressure can be related. Since the tank volume is fixed, the molar volume is related to the number of moles in the tank.

$T_A = T_A^i(V_A^i/V_A)^{R/Cv} = T_A^i(n_A/n_A^i)^{R/Cv} = 500[(601.4 - 0.1t)/601.4]^{2/5}$
When $t = 500$, $T_A = 482.9$ K, $P_A = P_A^i(T_A/T_A^i)^{Cp/R} =$
$0.5(482.9/500)^{7/2} = 0.443\text{MPa}$, $n_A^f = 601.4 - 50 = 551.4$ mols

Now, to find the temperature and pressure of Tank B, the energy balance could be solved. $n_B^fU_B^f - n_B^iU_B^i = \int H^{in}dn = \int H^{in}(dn/dt)\,dt$, where $H^{in} = Cp(T_A-T_R)$, and T_A is given above. T_A is a rather messy formula that would be best handled by numerical integration. It is much easier to draw a boundary around both tanks and realize that this system boundary results in an isolated system, choosing $T_R = 500\text{K}$,
$n_A^fU_A^f - n_A^iU_A^i + n_B^fU_B^f - n_B^iU_B^i = 0 =$
$551.4(482.9\text{-}500) - 601.4(0) + (252.6 + 50)(T_B^f - 500) - 252.6(300\text{-}500)$
where Cv has been cancelled from all terms. Simplifying,
$-9428.9 + 302.6\,T_B^f - 1.513\text{E}5 + 5.052\text{E}4 = 0$, $T_B^f = 364.2$ K
$P_B^f = nRT/\underline{V} = 302.6{*}8.314{*}364.2/7\text{E}6 = 0.131$ MPa

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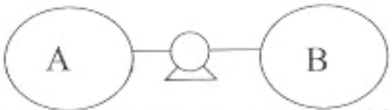
(4.39) Two storage tanks (0.1 m^3 each) contain air at 2 bar....



The questions are most easily answered in reverse order. Since the compressor is isothermal, $\Delta H = Q + W_S = 0$, and the heat and work must be decoupled. One way is to write a balance for isothermal work across the pump and integrate it. It is easier, however, to use entropy. If the process is reversible (no T gradients, no velocity gradients), then around the entire system,

$\Delta\underline{S} = \underline{Q}/T_{sys}$, $\Delta\underline{S} = n_A^fS_A^f - n_A^iS_A^i + n_B^fS_B^f - n_B^iS_B^i$, define reference state,
$T_R = 280\text{K}$, $P_R = 0.2$ MPa, this makes $S_A^i = 0$, and $S_B^i = 0$,
$S = Cp\ln(T/T_R) - R\ln(P/P_R) + S_R$
$n_B^f = P\underline{V}/RT = 0.3{*}0.1\text{E}6/8.314/280 = 12.887$ mols
$n_A^f = P\underline{V}/RT = 0.1{*}0.1\text{E}6/8.314/280 = 4.296$ mols
$S_A^f = -8.314{*}\ln(0.1/0.2) = 5.7628$, $S_B^f = -8.314{*}\ln(0.3/0.2) = -3.3710$
$\Delta\underline{S} = 4.296(5.7628) + 12.887(-3.371) = -18.687$ J/K
$\underline{Q} = T_{sys}\,\Delta\underline{S} = 280(-18.687) = \underline{-5232\text{ J}}$
Since the overall energy balance gives $\Delta\underline{U} = 0 = \underline{Q} + \underline{W}$, $\underline{W} = \underline{5232\text{ J}}$

(4.40) A constant pressure air supply is connected to a small tank (A) as shown...

a) $d\underline{U} = H^{in}dn$

$$ndU + Udn = H^{in}dn$$
$$\left(H^{in} - U\right)dn = ndU$$
$$\frac{dU}{H^{in} - U} = \frac{dn}{n};\ln\left(\frac{H^{in} - U^i}{H^{in} - U^f}\right) = \ln\frac{n^f}{n^i}\text{; now in terms of T}$$
$$H^{in} = C_P\left(T^{in} - T_R\right) + H_R$$
$$U = C_V\left(T - T_R\right) + U_R$$
$$H^{in} - U = C_PT^{in} - C_VT - \left(C_P - C_V\right)T_R + \left(H_R - U_R\right)$$

---

thus, $\dfrac{C_P T^{in} - C_V T^i}{C_P T^{in} - C_V T^f} = \dfrac{n^f}{n^i}$, rearranging

$$n^i\left(C_P T^{in} - C_V T^i\right) = n^f\left(C_P T^{in} - C_V T^f\right)$$

using $n = \dfrac{P\underline{V}}{RT}$

$$\frac{P^i}{T^i}\left(C_P T^{in} - C_V T^i\right) = \frac{P^f}{T^f}\left(C_P T^{in} - C_V T^f\right)$$

$$\frac{C_P}{C_V} = \gamma = 1.4$$

Solving for T^f; $T^f = \dfrac{T^{in}\gamma}{1 + \dfrac{P^i}{P^f}\left(\gamma\left(\dfrac{T^{in}}{T^i}\right) - 1\right)} = \dfrac{(292)(1.4)}{1 + \dfrac{1.013}{9}\left(1.4\left(\dfrac{292}{300}\right) - 1\right)} = \boxed{392.8\text{ K}}$

b) Reference state – initial conditions

$T_R = 300$ K, $P_R = 1.013$ bar, $S_R = 0$.

tank

$\underline{S}^i = n^i S^i = 0$ by choice of ref state

All T and P known, therefore S-balance is not required. Use P and T for find final entropy.

$\underline{S}^f = n^f S^f$; $n^f = \dfrac{P^f \underline{V}}{RT^f}$ = 9(1)/8.314E-5/393 = 275.5

$n^i = \dfrac{P^i \underline{V}}{RT^i}$ = 1.013(1)/8.314E-5/300 = 40.61 → $\Delta n_{tank} = -\Delta n_{surr} = 234.9$ mol

$$\Delta\underline{S}_{tank} = n^f S^f - \cancel{n^i S^i}$$

$$S^f = C_P \ln\frac{T^f}{T_R} - R\ln\frac{P^f}{P_R}$$

$$\Delta\underline{S}_{tank} = \frac{P^f \underline{V}^f}{RT^f}\left(C_P \ln\frac{T^f}{T_R} - R\ln\frac{P^f}{P_R}\right)$$

$$= \frac{9(1)}{\left(8.314\times10^{-5}\right)(393)}\left(29.3\ln\left(\frac{392.8}{300}\right) - 8.314\ln\left(\frac{9}{1.013}\right)\right) = \boxed{-2828.5\text{ J/K}}$$

surroundings

$d(nS) = -S^{out}dn^{out}$ integrates to

$\Delta\underline{S}_{surr} = S_{surr}\,\Delta n_{surr}$ $\quad S_{surr} = C_P \ln\dfrac{T_{surr}}{T_R} - R\ln\dfrac{P_{surr}}{P_R} = C_P \ln\dfrac{292}{300} - R\ln\dfrac{9}{1.013}$

$= -18.952$ J/molK; $\boxed{\Delta\underline{S}_{surr} = 4453.7\text{ J/K}}$

universe

$\Delta\underline{S}_{surr} + \Delta\underline{S}_{tank} = \Delta\underline{S}_{universe} = \boxed{1624\text{ J/K}}$

Note that $\Delta\underline{S}_{surr}$ and $\Delta\underline{S}_{tank}$ depend on the ref state, but $\Delta\underline{S}_{universe}$ does not.

c) $S^{out} > S^{in}$ because irreversible throttling. For a boundary around just the tank, the process is isentropic. ΔS = 0 = 29.3ln(T^f/392.8)-8.314ln(1.013/9) → T^f = 210.4K

This is a supplemental section not printed in the textbook. Suppose the process is cycled repeatedly. What happens to the temperatures?

TR 300 PR 1.013

Cycle	Pressurization											Depressurization
	Tank							Surroundings		Universe		Tank
	T(initial)	n(initial)	S(initial)	T(final)	n(final)	S(final)	$\underline{S}$tank	Ssurr	$\underline{S}$surr	$\underline{S}$univ		T(final)
1	300	40.61423	0	392.7672	275.6115	-10.26591	-2829.402	-18.95228	4453.734	1624.332		210.4249
2	210.4249	57.90316	-10.39135	369.5835	292.9004	-12.04853	-2927.328	-18.95228	4453.734	1526.405		198.0042
3	198.0042	61.53539	-12.17397	365.0564	296.5328	-12.40964	-2930.734	-18.95228	4453.734	1522.999		195.5789
4	195.5789	62.29848	-12.53509	364.1194	297.2957	-12.48495	-2930.805	-18.95228	4453.734	1522.929		195.0769
5	195.0769	62.4588	-12.61039	363.9232	297.456	-12.50074	-2930.792	-18.95228	4453.734	1522.941		194.9717
6	194.9717	62.49248	-12.62619	363.882	297.4897	-12.50406	-2930.788	-18.95228	4453.734	1522.945		194.9497
7	194.9497	62.49956	-12.6295	363.8733	297.4968	-12.50476	-2930.787	-18.95228	4453.734	1522.946		194.945
8	194.945	62.50104	-12.6302	363.8715	297.4983	-12.5049	-2930.787	-18.95228	4453.734	1522.946		194.9441

The spreadsheet shows that the cycle becomes 'steady' and T_{high} = 363.9K and T_{low} = 194.9 K.

(4.41) The pressurization of problem 4.40 is performed by replacing…

(a)Internally reversible, $dS_{gen} = 0$

Note that some type of heat transfer between the device and tank is necessary to keep the device outlet T and the tank temperature equal during the compression. $ndS + Sdn = S^{in}dn$

Set T_R, P_R (choose $S_R = 0$)

$$S^{in} = C_P \ln\frac{T^{in}}{T_R} - R\ln\frac{P^{in}}{P_R} + S_R \quad \text{all known}$$

Entropy Balance for reversible process:

$$\left(S^{in} - S\right)dn = ndS$$

$$\frac{dn}{n} = \frac{dS}{S^{in} - S}\text{; Integrating, } \frac{n^f}{n^i} = \frac{S^{in} - S^i}{S^{in} - S^f}\text{, (1); but also, } \frac{n^f}{n^i} = \frac{P^f}{T^f}\frac{T^i}{P^i}\text{, (2).}$$

Entropy can also be written in terms of P, T:

$$S^i = C_P \ln\frac{T^i}{T_R} - R\ln\frac{P^i}{P_R} + \overset{0}{\cancel{S_R}}\text{, (3); and } S^f = C_P \ln\frac{T^f}{T_R} - R\ln\frac{P^f}{P_R} + \overset{0}{\cancel{S_R}}\text{, (4).}$$

$P^f = 9$ bar, $\therefore T^f$ is only unknown by combining (1)-(4);

$$\frac{P^f T^i}{T^f P^i} = \frac{C_P \ln(T^{in}/T^i) - R\ln(P^{in}/P^i)}{C_P \ln(T^{in}/T^f) - R\ln(P^{in}/P^f)},$$

$$\frac{0.9(300)}{T^f(0.1013)} - \frac{29.3\ln(292/300) - 8.314\ln(0.9/0.1013)}{29.3\ln(292/T^f) - 8.314\ln(0.9/0.9)} = 0; \boxed{T^f = 315.2\text{K}}$$

then, $\frac{P^f V}{RT^f} = n^f$ = 0.9MPa(1E6cm^3)/8.314/315.2 = $\boxed{343.4 \text{ mol}}$, $\boxed{n^i = 40.61 \text{ mol}}$

(b) $U^f = C_V(T^f - T_R) - U_R$, $H^{in} = C_P(T^{in} - T_R) + H_R$

$H_R = U_R + (RT)_R$; set ref state to $H_R = 0 = H^i$; $U_R = -RT_R$ = -8.314(292) = -2428 J/mol

$n^f U^f - n^i U^i = H^i\left(n^f - n^i\right) + \underline{W}_S$, and by ref state the flow on the RHS drops,

$\underline{W}_S = n^f U^f - n^i U^i$ = 343.4(20.99(315.2-292)-2428)-40.61(20.99(300-292)-2428)
= $\boxed{-575 \text{ kJ}}$

Note that any other ref state will give same work but H^i term will be nonzero.

Note: Heat transfer will be necessary between tank and expander to assure expander outlet and tank remain at the same T.

(4.42) A 2 m^3 tank is at 292 K and 0.1 MPa and it is desired to pressurize….

a) Find: T^f, n^f

$$dnU = H^{in}dn^{in}$$

$$ndU + Udn = H^{in}dn$$

$$ndU = \left(H^{in} - U\right)dn$$

$$\frac{dU}{\left(H^{in} - U\right)} = \frac{dn}{n}$$

$$-\ln\frac{\left(H^{in} - U\right)}{\left(H^{in} - U^{i}\right)} = \ln\frac{n^f}{n^i};\ \frac{H^{in} - U^{i}}{H^{in} - U^{f}} = \frac{P^f}{T^f}\frac{T^i}{P^i}$$

$P\underline{V} = nRT$; $n = \frac{P\underline{V}}{RT}$ where $\underline{V} = 2$ m^3; $T^i = 292$ K, $P^i = 0.1$ MPa, $P^f = 3$ MPa

$$H^{in} = C_P\left(T^{in} - T_R\right) + H_R;\ U^i = C_V\left(T^i - T_R\right) + U_R;\ H^{in} - U^i = C_PT^{in} - C_VT^i$$

$$\frac{P^i}{T^i}\left(C_PT^{in} - C_VT^i\right) = \frac{P^f}{T^f}\left(C_PT^{in} - C_VT^f\right)$$

$C_V = (C_P - R);\ \frac{C_P}{C_V} = \gamma$ =29/20.7 = 1.4

Solving for T^f

$$T^f = \frac{T^{in}\gamma}{1 + \frac{P^i}{P^f}\left(\gamma\left(\frac{T^{in}}{T^i}\right) - 1\right)}$$

$$= \frac{(350)(1.4)}{1 + \frac{0.1}{0.3}\left(1.4\left(\frac{350}{292}\right) - 1\right)} = \boxed{399.7\text{ K}}$$

$$n^f = \frac{(0.3\text{MPa})(2E6\text{cm}^3)}{8.314T^f} = 180.6\text{ mol}$$

b) Incoming fluid different from tank T; Throttle valve irreversible;

c) Viscous dissipation; Thermal gradients; Heat transfer from tank walls

d) $dnS = S^{in}dn$

$$ndS + Sdn = S^{in}dn$$

$$ndS = \left(S^{in} - S\right)dn$$

$$\frac{dS}{S^{in} - S} = \frac{dn}{n}$$

Same integration form as part (a)

$$\frac{S^{in} - S^i}{S^{in} - S^f} = \frac{P^f}{T^f}\frac{T^i}{P^i}$$

T^f is only unknown

$$S^{in} = C_P\ln\frac{T^{in}}{T_R} - R\ln\frac{P^{in}}{P_R}$$

$T^{in} = 350$ K, $P^{in} = 5$ MPa

$S^i = f(T^i = 292 \text{ K}, P^i = 0.1 \text{ MPa})$

$S^f = f(T^f = ?, P^f = 3 \text{ MPa})$

e) Work would be involved since energy balance changes to balance at new T.

$dnU = H^{in}dn + d\underline{W}$

$n^f U^f - n^i U^i = H^{in}\left(n^f - n^i\right) + \underline{W}_S$

get T^f from (d), use equations of U, H from (a), calculate $\underline{W}$

(4.43) Two gas storage tanks…

$\Delta \underline{U} = 0 = \underline{Q} + \underline{W}_S \qquad \underline{W}_{EC} = 0$

$\Delta \underline{S} = \dfrac{\underline{Q}}{T}$

Initial State:

V_2 = 24775 cm^3/mol

n_2^i = (1 m3)(10^6 cm^3/m^3)(mol/2  75 cm^3) = 40.36 mol

V_1 = 825.8 cm^3/mol

n_1^i = 1210.9 mol

n_T = 1251.3 mol

Final State:

$P_1 = P_2$

$n_1 = n_2 = \frac{1}{2}n_T$ = 625.6 mol

$P^f = \dfrac{nRT}{\underline{V}} = \dfrac{(625.6)(8.314)(298)}{10^6}$

P^f = 15.5 bar

Reference State: 298 K, 1 bar

S_1^i = -28.2776 J/mol K

$S_2^i = 0$ (at reference state)

$S_1^f = S_2^f$ = -22.7873 J/mol K

$\Delta \underline{S}$ = (1251.3)(-22.7873) – (1210.9)(-28.2776) = 5727 J/K

$\underline{Q}$ = (5727)(298) = 1,706,807 J

$\underline{W}$ = -1706 kJ