Example The state of stress is given by $\sigma_x = 25p$ and $\sigma_y = 5p$ plus shearing stresses τ_{xy}. On a plane at 45° counterclockwise to the plane on which σ_x acts the state of stress is 50 MPa tension and 5 MPa shear. Determine the values of σ_x, σ_y, τ_{xy}.

Example The displacements of points in a deformed elastic solid (u) are related to the coordinates of the points (x) by a vector relationship $u_i = e_{ij}x_j$. Expand this tensor expression.

Example Strain-gage measurements made on the free surface of a steel plate indicate that the principal strains are 0.004 and 0.001 in/in. What are the principal stresses?

1. Determine the principal stresses for the stress tensors with rectangular Cartesian components given in the matrices below (units are kpsi). Determine also direction cosines for each principal axis of a right-handed system. *Hint*: The cubic equations are easily factored.

(a) $\begin{bmatrix} 18 & 0 & 24 \\ 0 & -50 & 0 \\ 24 & 0 & 32 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -10 & 0 \\ -10 & 0 & 30 \\ 0 & 30 & -27 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

2. Evaluate the three invariants I_σ, II_σ, III_σ for the stress tensors of Ex. 1 by using the given components and also by using the values obtained in that problem for the principal stresses, and check that the same results are obtained by the two procedures.

3. Separate each of the matrices in Ex. 1 into the sum of two matrices, representing the spherical and deviatoric stresses.

1. For each of the following displacement gradient matrices sketch the deformed position of an element which was initially a square in the XY-plane with sides parallel to the axes.

$$\begin{bmatrix} 0 & 0.01 & 0 \\ 0.01 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & -0.01 & 0 \\ 0.01 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0.02 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

2. For each of the following displacement gradient matrices J_u, determine the strain matrix, the rotation matrix, the volume strain, and the deviatoric strain matrix.

(a) $10^{-4}\begin{bmatrix} 9 & 10 & -4 \\ -10 & 18 & -18 \\ -14 & -18 & 27 \end{bmatrix}$ (b) $10^{-4}\begin{bmatrix} 4 & 1 & 4 \\ -1 & -4 & 0 \\ 0 & 2 & 6 \end{bmatrix}$

3. For each matrix J_u of Ex. 2, evaluate:
   (1) the three invariants I_ε, II_ε, III_ε of the strain
   (2) the three invariants $I_{\varepsilon'}$, $II_{\varepsilon'}$, $III_{\varepsilon'}$ of the deviatoric strain.

4. Show that the matrix operation Ωn gives the same unit