021004 Quiz 2 Mechanics of Materials

1) -For a mechanically isotropic material for which you know the tensile modulus and the shear modulus can you calculate Poisson's ratio?
   -Explain by describing the number of parameters in the modulus tensor.

2) Show that the area under the stress-strain curve is equal to the stored energy density. Do this by writing an expression for the change in energy as a function of the force and displacement and through an integral of this expression.

3) Calculate τ_{11}, τ_{12} using the following matrix equation.

$$\begin{bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{bmatrix}$$

   -Give expressions for the tensile (E) and shear (G) modulus.

4) For a creep measurement plot x (displacement) versus t (time) for a Hookean elastic, a Newtonian Fluid and an anelastic material showing Hookean, Newtonian and relaxation behaviors.
   -By writing two linear constitutive equations for Hookean and Newtonian behavior show that G/η has the units of time, the viscoelastic relaxation time.
   -Give your opinion on if this time, $\tau_{VE} = G/\eta$, could differ from the relaxation time associated with anelastic relaxation behavior shown in the first part of this question?

Answers: 021004 Quiz 2 Mechanics of Materials

1) For a mechanically isotropic material there are only two independent mechanical constants. That is, if you are given any two mechanical constitutive parameters such as G and E, you can calculate all other parameters such as ν.

2) This is from the notes:

> The relationship between strain energy and stress and strain can be obtained by considering that energy is equal to the applied force, F, times the change in distance over which the force acts,
>
> $$dU = F(x)dx = xF(\varepsilon)d\varepsilon \qquad (33)$$
>
> where x is a unit length. The force in the first graph of Figure 4 follows the function,
>
> $$F(\varepsilon) = AE\varepsilon = x^2 E\varepsilon \qquad (34)$$
>
> where A is the unit area associated with stress. Then we have,
>
> $$dU = F(x)dx = V_{unit}E\varepsilon d\varepsilon \qquad (35)$$
>
> where V is a unit volume. Dividing the energy by the unit volume provides the strain energy density, U_0. Integration of equation (35) yields,
>
> $$U_0 = \int_{\varepsilon=0}^{\varepsilon} E\varepsilon d\varepsilon = \frac{1}{2}E\varepsilon^2 = \frac{1}{2}\sigma\varepsilon \qquad (36)$$

3)

$$\tau_{11} = 2\mu\varepsilon_{11} + \lambda(\Delta) = E\varepsilon_{11}$$
$$\tau_{12} = 2\mu\varepsilon_{12} = G\varepsilon_{12}$$
$$G = 2\mu$$
$$E = 2\mu + \frac{\lambda(\Delta)}{\varepsilon_{11}}$$

4)

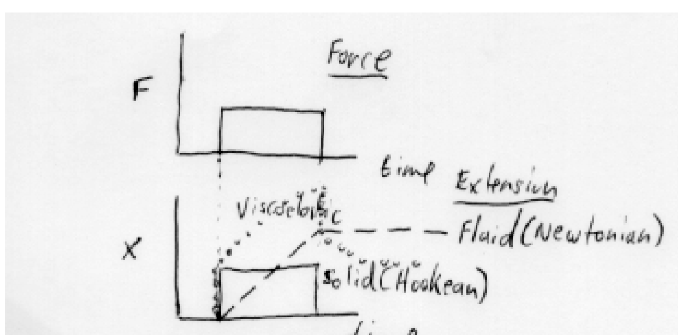

Figure 3. Creep Measurement for Solid —; Liquid(- - -); and Viscoelastic (.....)

$$\tau = G\gamma \qquad \textbf{so} \qquad \mathbf{G} = \frac{\tau}{\gamma}$$

$$\tau = \eta \frac{d\gamma}{dt} \qquad \textbf{so} \qquad \eta = \frac{\tau}{d\gamma/dt}$$

$$\frac{G}{\eta} = \frac{d\gamma/dt}{\gamma} = \tau_{VE}$$

The two relaxation times are not the same. The anelastic relaxation time can be independent of viscoelastic relaxation time. There is not correct answer to this for the purpose of the quiz as long as you give some reasonable answer.