021021 Quiz 3 Mechanics of Materials

1) We discussed the Cauchy tensor and the Finger tensor.
   -How are these two tensors related to each other?
   -Explain the need for these tensors relative to infinitesimal strain, ε, and vorticity, ω, tensors in terms of:
   -Large strains.
   -Normal stresses
   -Molecular or other thermodynamic models for the free energy

2) The Cauchy strain tensor is defined by,

$$C_{jk} = \frac{\partial r_i'}{\partial r_j}\frac{\partial r_i'}{\partial r_k}$$

   explain the terms in this tensor, r and r', and δr and δr'.
   -What is the reference state for the Cauchy tensor and why?
   -What is the reference state for the Finger strain tensor?

3) -Calculate the Cauchy tensor for tensile strain of an incompressible material where the z (3)-elongation is λ (remember that $\lambda_1\,\lambda_2\,\lambda_3 = 1$ and $\lambda_1 = \lambda_2$).
   -For this diagonal matrix do a matrix transformation to obtain the Finger Strain Tensor.
   -Calculate the Finger tensor from the elongations in the 1, 2 and 3 directions.

4) -What is the difference between true stress and engineering stress?
   For an elastomer under tensile stress the true stress is given by,

$$\sigma_{11} = \rho kT\left(\lambda^2 - \frac{1}{\lambda}\right)$$

   -Does this relate better to the Cauchy strain tensor or the Finger strain tensor of question 2?

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1)

$$\underline{\underline{B}} = \frac{1}{\underline{\underline{C}}}$$

Finger and Cauchy strain tensors are needed to describe infinitesimally small material elements in terms of strains that lead to changes in the free energy. These small elements can be subject to torsion and translation even when the bulk sample is not. The strain and vorticity exclude these same strains for the bulk sample but do not do this for small material elements.
The previous description of strain and vorticity are for small strains only. The Cauchy and Finger tensors can describe any strain.
The Cauchy and Finger tensors can describe normal stress differences since they include descriptions of how material elements are interconnected on a small scale. Vorticity and strain tensors can not do this.
Only a local description of strain such as found in the Cauchy or Finger tensors can be used to model materials using a free energy expression.

2) r refers to a point in the deformed state and r' refers to a point in the undeformed state. δr refers to the difference vector between two points in the deformed state and δr' refers to the distance vector between two points in the undeformed state.
The reference state is the deformed state since this is akin to a real strain.
The reference state for the Finger tensor is the undeformed state.

3)

$$\underline{\underline{C}} = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \frac{1}{\lambda^2} \end{pmatrix} \text{ and } \underline{\underline{B}} = \begin{pmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & \lambda^2 \end{pmatrix}$$

$$B_{ij} = \Sigma\, \delta r'_i/\delta r_\mu\, \delta r'_j/\delta r_\mu$$

4) True stress is calculated based on the area of the elongated sample, engineering stress is caluculated based on the area of the undeformed sample
The strain expression for an elastomer relates directly with the Finger tensor.