021121 Quiz 5 Mechanics of Materials

1) Ductile yielding in materials is generally described by one of two models.

a) Sketch stress versus strain to show the yield stress and the general behavior of a ductile material such as copper or aluminum.
b) What is the deviatory stress and why is it used to describe yielding?
c) Give the equation for the Von Mises' criterion and explain its relationship in general terms to the deviatory stress.
d) Give the equation for the Tresca criterion and explain its relationship in general terms to the deviatory stress.

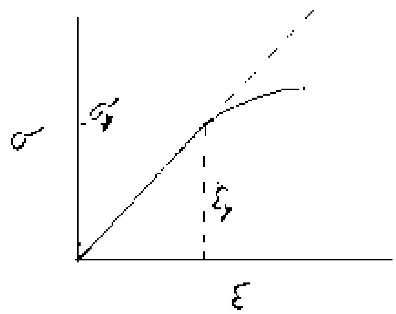
2) The plane stress condition is used as the simplest case for derivation of yielding and fracture equations
a) Define plane stress conditions and give an example of a mechanical deformation where plane stress conditions are expected.
b) Define plane strain conditions and give an example of a mechanical deformation where plane strain conditions are expected.
c) Why do you think plane stress is a simpler mechanical system to analyze?
d) If a plastic deformation mechanism depended on dilatational stress, would it most likely be observed under plane stress or under plane strain? (Cavitation is an example of such a deformation mechanism.)

3) The seminal equation to describe failure in materials is the Griffith equation.
a) Describe the type of material for which the Griffith equation was derived.
b) Give a plot of stress versus strain to show this type of material with an "x" showing the point of failure.
c) Give the Griffith equation.
d) If atomic models are used an equation can be obtained that has the same functional form as the Griffith equation. What is the approximate difference in failure stress between the Griffith model and this atomic model?
e) What morphological feature accounts for this difference?

4) The derivation of the Griffith equation involves calculation of the strain-energy involved in elastic deformation of a sample prior to yielding. This lead to the quantification of fracture toughness in terms of G_I, K_I, and G_{Ic} and K_{Ic}.
a) Define these four values.
b) Give a relationship between G_{Ic} and K_{Ic}.
c) If the native flaws in a material could be forced to be spherical would the value of K_{Ic} change? Why?

Answers: 021121 Quiz 5 Mechanics of Materials

1) a)



b) $\underline{\sigma}' = \underline{\sigma} - P\underline{\delta}$

The deviatory stress is used to describe ductile yielding since it is assumed that hydrostatic pressure does not effect yielding.

c) The Von Misses' criterion considers that the second invariant of the deviatory stress must reach a critical value for yielding to occur,

$$J_{2\varepsilon} = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\right] = k^2$$

where the first function is for the principle axes.

d) The Tresca criterion considers that the shear stress reaches a critical value for yielding to occur. The shear stress depends on the second and third invariants of the deviatory stress. The Tresca criterion is given by,

$$\tau^* = k = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_0}{2}$$

where the latter equations are written for the principle stresses and for, plane stress and uniaxial tension conditions respectively.

2) a) Plane stress is expected for a thin sheet in uniaxial tension. In plane stress there are no dilatational stresses, i.e. stresses normal to the tensile direction can completely relax.

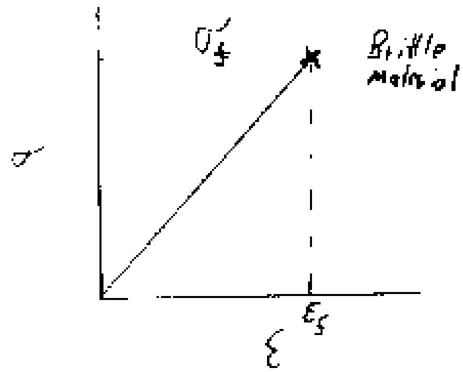
b) Plane strain occurs when stresses normal to the tensile direction can no relax at all such as when an infinite bulk sample is considered under uniaxial stress.

c) In plane stress we do not have to consider dilatation.

d) Cavitation and other dilatational deformations will not be observed under plane stress conditions. They will, most likely, be observed under plane strain conditions where significant dilatational stresses are likely to occur.

3) a) The Griffith equation is derived for a brittle material.

b)



c) $\sigma^* = \left(\frac{E(2\gamma_s)}{\pi c}\right)^{1/2}$ **plane stress conditions**

d) Atomic models show is 10 to 1000 times the failure stress of the Griffith model.

e) Atomic models are based on atomic dimensions for c (and a spherical flaw shape), the Griffith equation are based in innate flaws in all brittle materials, c is on the order of 1 to 10 µm in size (and having a linear shape with an atomically small crack or flaw tip).

4) a) G_I is the fracture toughness, or strain-energy release rate, for mode I fracture (tensile mode)
G_{Ic} is the critical value of fracture toughness, i.e. the value of G_I at fracture.

$$G_{Ic} = \frac{\pi a \sigma_c^2}{E}$$

where a is half the native flaw length,σ_c is the bulk stress at failure and E is the tensile modulus.
K_I is the value of the stress intensity factor associated with a native flaw in the tensile mode.
K_{Ic} is the critical value of the stress intensity factor at failure. Using the model of a native flaw with parallel flaw lines, an infinitesimally narrow gap and a flaw tip radius of atomic dimensions,

$$K_{Ic} = \sigma(\pi a)^{1/2}$$

where a is half the flaw length.

b) $K_{Ic}^2 = G_{Ic} E$

c) K_{Ic} depends on the model used to describe the crack since it is a stress intensity factor. G_{Ic} does not depend on the crack model.