# Quiz 3, Polymer Processing, 2001

The *Reynold's Equation*,

$$v_x(y) = \frac{1}{2\eta}\frac{\partial P}{\partial x} y(y-H) + V_0 \frac{y}{H}$$

is obtained from the *Navier-Stokes Equation* applied to simple shear flow between two parallel plates. Only the x-component of the *Navier-Stokes Equation* was used to obtain $v_x(y)$ in class,

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \eta\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x \quad \textbf{(x-part of NSE)}$$

a) Many of the terms in the Navier-Stokes equation can be set to 0 by simple assumptions. **-List** (with reference to the effected term) the assumptions involved in obtaining the simplified x-component of the Navier-Stokes equation given below.

$$\frac{\partial P}{\partial x} = \left(\frac{\partial \tau_{xy}}{\partial y}\right) = \eta\left(\frac{\partial^2 v_x}{\partial y^2}\right)$$

b) Through integration and application of limits a simple description of the x-velocity is obtained,

$$v_x(y) = \frac{1}{2\eta}\frac{\partial P}{\partial x} y(y-H) + V_0 \frac{y}{H}$$

**-Sketch** the x-velocity profile as a function of y for the two terms in this expression
**-as well as** for the entire Reynold's Equation under an applied pressure gradient and a non-zero plate velocity.
**-Try to give** an analogous situation for heat conduction where two similar terms might be involved and explain. (Hint: Compare Fourier's law, $q_y = k\ dT/dy$ with $\tau_{xy} = \eta\ dv_x/dy$ for equivalent parameters.)

c) **-Rewrite** the equation of question "b" using the Reynold's number, Re.

d) Re = 2000 is considered a critical value for fluid flow.
**-What** behavior is observed for Re>2000? (Give and example.)
**-Is** Re usually above or below 2000 in polymer processing. Why?
**-Why** would flow in a thin gap filled by oil have any similarity to flow of a polymer in a large gap such as in an extruder?

e) In addition to the Reynold's number there are a number of other dimensionless groups used in polymer rheology. One we have mentioned is the Deborah number. Another is the Weissenberg number, We = $\psi_1/\tau_{12}$, where $\psi_1$ is the first normal stress difference, $\psi_1 = (\tau_{11}-\tau_{22})$, and $\tau_{12}$ is the shear stress. An empirical constitutive equation for the first normal stress difference is $\psi_1 = \Psi_1\ (d\gamma/dt)^2$, where $\Psi_1$ is a constitutive parameter (a constant).
**-Use** a simple constitutive equation for $\tau_{12}$ to obtain an expression for "We".
"We" has been used to estimate the number of entanglements in a polymer melt.
**-Explain** why "We" might be used in this way.

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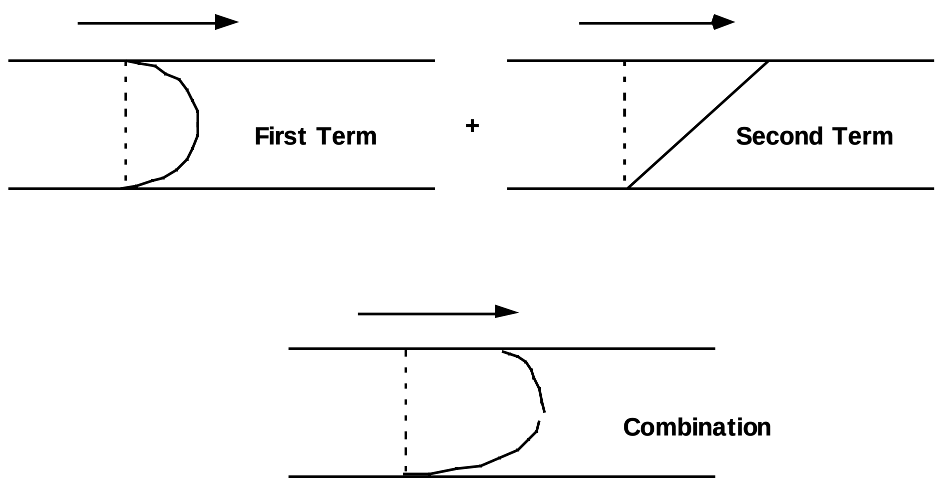
a.) Steady State- First term is 0
Flow only in x direction 3'rd and 4'th term 0
No elongational flow, second term is 0, on right hand side 2'nd term is 0
Infinite plates (no change in x velocity in z direction) 4'th term on RHS is 0
No gravity effects, last term 0.

The equation in terms of shear stress depends on Newton's constitutive equation.

b.)




In heat conduction comparison of Fourier's law with Newton's viscosity law indicates that an analogous situation would be the temperature profile for heat flow between two parallel plates. The pressure term would be equivalent to uniform production of heat in the gap such as by resistive heating while the second, linear term would be equivalent to the temperature profile resulting form a temperature difference between the two plates.

c.) Re = $H\rho V_0/\eta$
so,

$$v_x(y) = \frac{1}{2\eta}\frac{\partial P}{\partial x} y(y-H) + V_0 \frac{y}{H}$$

becomes,

$$v_x(y) = \mathrm{Re}\left\{ \frac{y}{2\rho V_0}\left( {}^{y}\!/_{H} - 1 \right)\frac{\partial P}{\partial x} + \frac{\eta y}{\rho H^2} \right\}$$

d.) For Re>2000 flow is turbulent. Re is **always** below 2000 for polymer processing. Turbulent flow is difficult to control and would lead to poor processed materials in almost any application. Flow in a thin gap for a low viscosity fluid like oil is similar to flow of a polymer in a wide gap because they have similar Reynold's number. An example of turbulent flow is the formation of shark skin and melt fracture in an extruder.

e.) We = $\Psi_1/\eta\ (d\gamma/dt)$. This could be used to estimate entanglements since the presence of entanglements and orientation of chains causes normal stress differences to occur. We = 0 means no entanglements, We = high means many entanglements. The equation indicates that entanglements are associated with high rates of strain and a high ratio between the first normal stress coefficient and the shear viscosity.