030128 Quiz 2 Polymer Processing

1) a) What is the difference between a constitutive equation and an equation of continuity?

b) The normal stress difference, $\psi_1 = \tau_{22} - \tau_{11}$, is found to be proportional the square of the rate of shear strain, $\dot{\gamma}_{12}$. This is described with the equation:

$$\psi_1 = \Psi_1 \dot{\gamma}_{12}^2 \quad \textbf{where } \Psi_1 \textbf{ is a constant}$$

Is this a continuity equation?

c) Why might the normal stress difference be related to die swell?

d) Does knowing this equation imply a physical or mechanistic understanding of the process that leads to die swell?

2) a) In the extruder lab we wrote $Q = Q_d + Q_p$. Is this a continuity equation?

b) Sketch the velocity profile associated with Q_d, Q_p and Q in the extruder channel assuming parallel plates and a Newtonian fluid. (Note the 0 velocity line.)

c) For the following flow geometry,

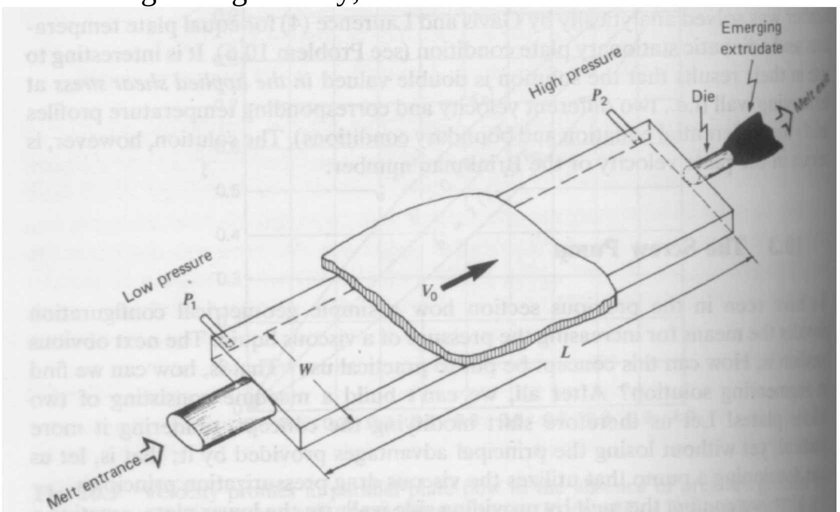

Fig. 10.8 A finite length shallow channel covered with an infinite moving plate.

Tadmor gives,

$$Q = \frac{V_0 W H}{2} + \frac{W H^3}{12 L \mu}(P_1 - P_2)$$

rewrite this equation to calculate the pressure drop (P_1 - P_2).

d) Take the derivative of this expression to obtain the channel depth for the maximum pressure drop, H_{max}, as a function of Q, W and V_0.

3) a)Write the total stress tensor and its two component tensors noting the number of independent terms in each tensor.

b) Write an expression that relates these three tensors

c) Write the total velocity gradient tensor and its two component tensors noting the number of independent terms in each tensor.

d) Write an expression that relates these three tensors.

ANSWERS: 030128 Quiz 2 Polymer Processing

1) a) A constitutive equation relates a pertubation to a response through a constitutive parameter such as the equation for shear stress in terms of the viscosity and rate of strain.
A continuity equation is a balance equation. Equations of continuity in mass, momentum and energy are commonly used to model polymer processing operations.
b) The equation for the first normal stress difference is not an equation of continuity.
c) Die swell occurs in response to a normal stress in the "22" direction.
d) Constitutive equations such as that given for the first normal stress difference are just empirical predictive equations and do not reflect understanding except in terms of predicting a specific response.

2) a) The equation for mass flow rate is a continuity equation.
b) From Tadmor and Gogos

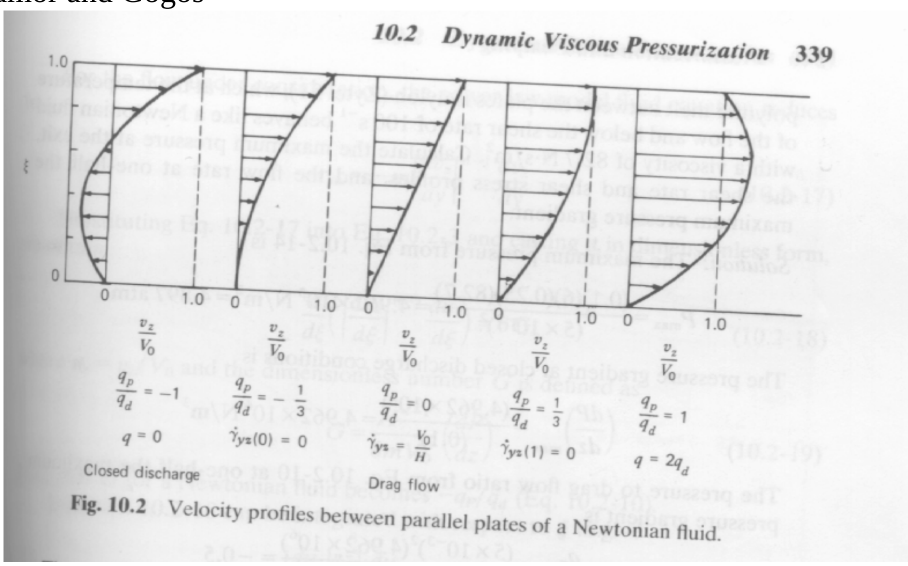

Fig. 10.2 Velocity profiles between parallel plates of a Newtonian fluid.

c)

$$(P_1 - P_2) = \frac{6L\mu}{H^2}\left(\frac{2Q}{WH} - V_0\right)$$

d)

$$\frac{d(P_1 - P_2)}{dH} = 12L\mu\left(\frac{Q}{WH^3} - \frac{V_0}{2H^2}\right)$$

setting $\frac{d(P_1 - P_2)}{dH} = 0$ at H_{max}, and solving for H_{max},

$$H_{max} = \frac{3Q}{WV_0}$$

3) a and b)

$$\pi = \begin{pmatrix} P+\tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & P+\tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & P+\tau_{33} \end{pmatrix}$$

$\pi = P\delta + \tau$

6 1 5 Components.

c and d)

$$\nabla v = \begin{pmatrix} \frac{\partial v_1}{\partial x_1} & \frac{\partial v_2}{\partial x_1} & \frac{\partial v_3}{\partial x_1} \\ \frac{\partial v_1}{\partial x_2} & \frac{\partial v_2}{\partial x_2} & \frac{\partial v_3}{\partial x_2} \\ \frac{\partial v_1}{\partial x_3} & \frac{\partial v_2}{\partial x_3} & \frac{\partial v_3}{\partial x_3} \end{pmatrix}$$

Del v = 1/2(dγ/dt +del ω)

9 6 3 Components

del ω is the difference between **del v** and its transpose.

the rate of strain is the sum of **del v** and its transpose.