020327 Properties Quiz 1

a) Flory discusses a Gaussian coil in his book *Polymer Chemistry*.
-Give the Gaussian distribution function and explain how it pertains to a polymer chain.

b) Give a value for $<R>$ and $<R^2>$ for a Gaussian chain.
How can these be obtained from the Gaussian distribution function?

c) What are the mass fractal dimensions for a Gaussian chain and a plate.
How can these two objects be distinguished from a dimensional perspective?

d) The characteristic ratio, C_n, is the ratio of the actual end-to-end distance to that of a Gaussian chain as calculated on the basis of a chemical bond where, n = number of bonds and b = the bond length. A typical value is 4 for polystyrene with n=1000.
Explain how long range and short range interactions might lead to values of $C_n>1$.

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a) $$P_G(R,n)\,dR = 4\pi R^2\left(\frac{2}{3}\pi nb^2\right)^{-3/2}\exp\left[-\frac{3}{2}\left(\frac{R}{b\sqrt{n}}\right)^2\right]dR$$

b is the step size (persistence length) and n is the number of persistence lengths in the chain. $n^{1/2}b$ is the standard deviation for the Gaussian function and is equivalent to the RMS end-to-end distance for the polymer chain, $\langle R^2\rangle^{1/2}$.

b) $\langle R\rangle = 0$ and $\langle R^2\rangle = nb^2$

These are obtained from the Gaussian distribution function by:

$$\langle R\rangle = \int_0^\infty R P_G(R,n)\,dR = 0$$

$$\langle R^2\rangle = \int_0^\infty R^2 P_G(R,n)\,dR = nb^2$$

(limits should be -∞ to ∞)

c) Both a plate and a Gaussian chain have a mass fractal dimension of 2. The Gaussian chain is a linear chain so the connectivity dimension is 1 while the plate is a regular object so the connectivity dimension is d_f, i.e. 2.

d) Short range interactions (chain rigidity) gives the chain a longer persistence length than a chemical bond. Objects that are more linear have a larger end-to-end distance since a linearly stretched chain has the maximum end-to-end distance. The maximum characteristic ratio is $C_{n,max} = nb/(n^{1/2}b) = n^{1/2} = 32$ for a chain with n = 1000.
Long range interactions involve the exclusion of one part of the chain from the volume occupied by another part of the chain at long separation distances along the chain. The volume of the chain must naturally increase due to this excluded volume and this would result in a larger end-to-end distance. Excluded volume effects the mass fractal dimension of the chain as will be shown in class. Long range interactions are generally only observed when a chain is in a dilute or semi-dilute solution.