**020509 Quiz 7 Properties**

1) In the RPA equation, it is the coupling of concentration fluctuations that leads to the scattering being represented by a harmonic mean, HM = N/(Σ{1/Value}).

**a) Write** the RPA equation for a binary blend with thermal interactions.

**b) What** term in the RPA calculation accounts for the coupling of A and B concentration fluctuations?

The RPA equation accounts for the interpenetration of two chain structures and the effect on scattering by weighting the weaker scatterer to a greater extent.

**c) Show** this is true by calculating the harmonic mean for the two values 1 and 10.

**d)** The weaker scatterer shows the smaller concentration fluctuations,
**why would** the smaller concentration fluctuations dominate the mutual response from a mixed system?

**e) Give** an example of an electronic or mechanical system where the weakest component dominates the response?

2) **a) Write** the spring constant for an ideal chain
**and an expression** relating force to extension.

**b) Explain** the dependence of the force on temperature by relating the ideal chain to an ideal gas.

**c) Explain** the terms in the following equation:

$$A = n_c \int dr \int_0^{\infty} dN\, \Psi(r, N) \frac{3kT}{2 N l_p^2} r^2 + K$$

**d) Define** "affine" deformation.

**e) Define** the "deformation gradient tensor", $E_{ij}$.

**Answers: 020509 Quiz 7 Properties**

1)

a) $\frac{1}{\alpha_k^0} = \frac{1}{\alpha_k^{AA}} + \frac{1}{\alpha_k^{BB}} - \frac{2\chi kT}{v_c}$

b) $\underline{\Psi}_k$, the internal field.

c) (1/1 + 1/10)*2 = 1.8 so the harmonic mean is weighted towards the lower value.

d) This is a difficult question. It would appear that larger fluctuations are dissipated through the internal field to the smaller fluctuations. Consider the equation for the internal field,

$$\underline{\psi}_k = \frac{\psi_k \alpha_k^{AA}}{\alpha_k^{AA} + \alpha_k^{BB}}$$

so if AA fluctuations are relatively large the internal field is the same as the external field. If BB fluctuations are relatively large the internal field is weak. Since BB fluctuations respond only to the internal field this is self damping, i.e. for a big BB fluctuation the field is weak so the contribution to the response is small. Smaller fluctuations are better coupled to the system response. There may be much better answers to this question.

e) Parallel resistors where the smaller resistor dominates the resistance or a composite of weak and strong components in series for a tensile test where the mechanical response is dominated by the weaker components.

2)

a) $\underline{F} = \frac{-3kT}{Nl^2} \underline{r}$ **so** $k_{spring} = \frac{3kT}{Nl^2}$

b) The force is proportional to the temperature which is a signature of an entropically controlled system. In an ideal gas the gas molecules move with a kinetic energy that is proportional to kT so the energy imparted to the walls of a vessel is proportional to T, i.e. P = nkT/V. If the gas molecules could be connected along a linear string they would pull at the ends with an energy kT. This is basically a description of an ideal polymer chain.

c) $n_c$ is the number of chains in a rubber sample
N is the degree of polymerization of a chain in the sample
$\psi$ is the probability of an N length chain having an end to end distance of "r".
$l_p$ is the Kuhn length
K is an enthalpic constant.

d) Affine means that the chains within a rubber sample deform to the same extent and in the same direction as the bulk sample.

e) $E_{ij} = dR'_i/dR_j$, where the prime indicates the deformed state and i and j refer to direction indicies, 1 is x, 2 is y, 3 is z for instance.