060526 Quiz 9 Polymer Properties

This week we considered the basic approach to describe mechanical dynamics in polymers which includes the concepts of *anelasticity* and *linear response*.

1) Mechanics is based on constitutive equations and constitutive constants. A constitutive equation is generally empirical, meaning that it *has no theoretical basis* and *merely describes a series of observations*. You should be able to recognize a constitutive equation and to realize that there is no deeper understanding involved than the description of a simple response to a perturbation. (*In a sense constitutive equations are just a language to describe observations.* To quote [Oliver Heaviside](), "*Why should I refuse a good dinner simply because I don't understand the digestive processes involved?*") Constitutive equations form the basis of engineering and technology.

a) We can write that the response x(t) to a constant field, ψ_0, is given by $x(t) = \psi_0\ \mu(t)$ in a impluse experiment. Is this a constitutive equation? Explain.

b) For an ideal gas we know that PV = nRT. Is this a constitutive equation? Explain.

c) Bragg's law states that $d = 2\pi/q$, is this a constitutive equation?

d) When you turn the steering wheel on your car, the car turns in proportion and in the direction you turn in a one-to-one relationship in the limits of the play in the wheel and the fully turned steering. Is this relationship a constitutive relationship? Explain.

e) Newton's law of viscosity states that the shear stress on a fluid is proportional to the rate of strain (or velocity gradient). If a fluid obeys this proportionality what could we say concerning the molecular make-up of the material? What structural model does this law rely on?

2) In polymer mechanical dynamics we consider three main categories of deformation.

a) List these three categories and explain briefly what is involved.

b) Consider an analogous electrical system such as a dielectric subjected to an electric field resulting in a polarization of the material. Explain the three analogous types of perturbation in part "a" for an electrical system.

c) For a tensile creep measurement sketch a plot of the time dependent strain for a Newtonian fluid, a Hookean elastic and an anelastic material such as the "Silly Putty" shown in class.

d) The decay of the anelastic component of an anelastic material in creep follows

$(1 - \exp(-t/\tau))$. Explain what τ is and how it might be related to material properties.

e) Any dynamic process involves a frequency, ω_0, or strain rate $d\gamma/dt$. For instance the vibrations of a rocket engine are high frequency while mechanical strains on a rocket casing could also be of extremely low frequency due to wind shear for instance. How could this impact the design of an elastomeric rocket engine seal? (Use "τ" in your answer.)

3) Linear response theory is based on two main principles, *Causality* and *Superposition*.

a) Explain how causality and superposition are involved in the following function,

$$x(t) = \int_{\psi(t=-\infty)}^{\psi(t=t)} \alpha(t-t')\,d\psi(t')$$

where $\alpha(t)$ is the susceptibility.

b) Explain how causality and superposition are involved in the following function,

$$\psi(t) = \int_{x(t=-\infty)}^{x(t=t)} a(t-t')\,dx(t')$$

where a(t) is the time dependent modulus.

c) What is the "response function", $\mu(t)$?

d) Write a constitutive equation for the strain x(t) as a function of stress $\psi(t)$ using this function.

e) Give a sketch of the response function as a function of time for a damped harmonic oscillator (like a bell), a Hookean elastic, a Newtonian fluid and an anelastic material (polymer like "Silly Putty").

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1) a) This is a constitutive equation with the response function as a constitutive function. (It is possibly a non-linear constitutive equation.)
b) The ideal gas law is not a constitutive equation. There is no constitutive constant and it is an exact function that can be proved from basic concepts.
c) Bragg's law is not a constitutive equation. It is an exact function that can be proven to describe wave interference.
d) Yes, we rely on a constitutive relationship to drive a car.
e) A constitutive equation like Newton's viscosity law does not indicate anything concerning the make-up of the material, for instance sand grains will obey Newton's viscosity law even though they are composed of macroscopic particles rather than molecules. No structural model is implied by this law.

2) a) Creep where a constant stress is applied and the strain is followed as a function of time. Stress relaxation where a constant strain is applied and the decay of stress in time is followed. Dynamic mechanical test where an oscillating strain with a fixed strain amplitude is applied and the stress is followed as a function of time.
b) Creep would be analogous to a constant field applied to a dielectric, dielectric relaxation. Stress relaxation would be analogous to the decay of polarization with release of an electric field. DMA would be analogous to dynamic dielectric measurements. Also, DMA could be analogous to the interaction of electromagnetic radiation (light) with a material that scatters or diffracts since electromagnetic radiation consists of an oscillating electric field (coupled to an oscillating magnetic field).
c)

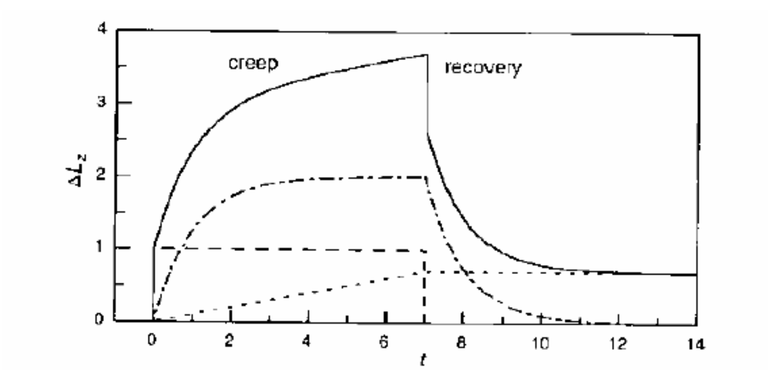

Fig. 5.1. Creep curve of a polymer sample under tension (schematic). The elongation ΔL_z induced by a constant force applied at zero time is set up by a superposition of an instantaneous elastic response (*dashed line*), a retarded anelastic part (*dash-dot line*) and viscous flow (*dotted line*). An irreversible elongation is retained after an unloading and the completion of the recovery process

d) τ is the relaxation time. It corresponds with approximately the time for the material to relax 1/3 of the total anelastic strain that will be recovered after infinite time. τ should be compared with the time of the process such as the strain rate or frequency. For the rubber balls in class, τ was near the "bounce time" for the dead ball and was a much longer time for the bouncy ball.

e) When a polymer component is expected to have constant properties across a wide range of temperature or frequency problems will be encountered. This was the case, for example, with the first space shuttle disaster where seals that were suitable at 40 C turned to glass at 32 C during launch allowing for a blow-by and destruction of the crew and the 30 billion dollar space craft. The destruction was determined to be due to an engineering miscalculation involving the behavior of τ by a congressional commission.

3) a) Casuality basically means that there is a constitutive equation that applies chronologically with the perturbation followed by the response. Superposition means that events sum in time modified by some type of memory function. In this case the memory function is the time dependent susceptibility, the perturbation is the stress and the response is the strain. A sequence of stress steps of infinitely small size are summed and weighted by the susceptibility to yield the strain at time "t".

b) For the equation for the time dependent stress in a stress relaxation measurement steps of strain are integrated using the time dependent modulus as a weighting factor.

c) $\mu(t) = d\alpha/dt$. It is the response to a impulse stress. The response function is also the memory function for stress in time,

$$x(t) = \int_{(t=-\infty)}^{(t=t)} \mu(t-t')\psi(t')dt'$$

d) See above.

e)

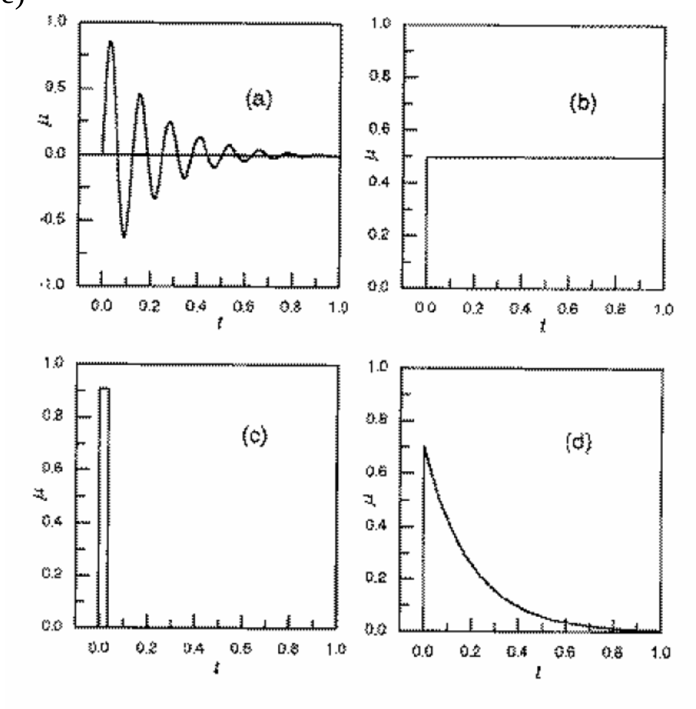

Fig. 5.4. Primary response function of a damped harmonic oscillator (*a*), a perfectly viscous body (*b*), a Hookean solid (*c*), a simple relaxatory system (*d*)

For Silly Putty, the response would be partly "d" above but with a small spike like "c" and a partial permanent deformation like "b".