080119 Quiz 2 Polymer Properties

Polymer chains that follow a random walk can be described by a fractal scaling law where the chain size, R, scales with the mass or degree of polymerization, z.

1) a) Write a general scaling law between mass and size. Name the scaling exponent associated with this scaling law.
b) What values would you expect this exponent to display for i) a rod, ii) a Gaussian chain, iii) a self-avoiding walk, iv) a sheet of paper and v) a crumpled sheet of paper that is 75% air?
c) Do any of these exponents have the same value? Explain how you could describe the differences between these structures in this case.

2) Balankin et al. (*Phys. Rev. E* **75** 051117 (2007)) describe the scaling properties of randomly folded balls of aluminum foil (left below). They determine density versus size plots (right) where h is the sheet thickness and R is the ball diameter.

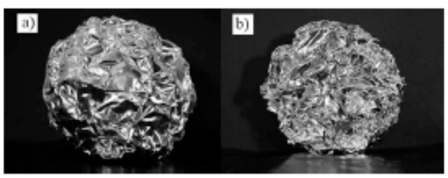


FIG. 1. Images of (a) balls folded from an aluminum sheet of thickness h=0.06 mm and edge size L=60 cm and (b) the cut through this ball.

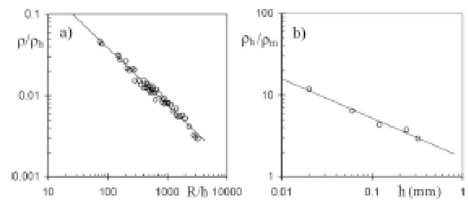


FIG. 3. (a) Data collapse for ρ/ρ_h versus R/h (the slope of the fitting line is $3-D$=0.7009, R^2=0.98); and (b) log-log plot of ρ_h/ρ_m versus h (straight line is given by $y=1.728x^{-0.4816}$, R^2=0.98).

a) For the scaling law you propose in 1a) how do you expect density to scale with size? If the slope in the first plot (a log-log plot of reduced density versus reduced size) above is -0.7 what is the scaling exponent from question 1a? Does this agree with your values of question 1b)?
b) What is the connectivity dimension for crumpled aluminum foil?
c) The tensile modulus for a crumpled sheet, E, is related to the minimum path through $E = E_{sheet} (R/a)^{-(3+dmin)}$ where R is the ball size and a is the size of the folds that make up the crumpled sheet (Witten *J. Phys. II France* **3** 367-383 (1993)). E_{sheet} is the modulus for the uncrumpled sheet. Explain why the modulus associated with straightening out the crumpled ball would be related to the minimum path and the minimum dimension.

3) a) Describe the difference between short range interactions (SRI) and long range interactions (LRI). For the balled sheets of question 2, consider that balls of paper spring open when pressure is released while balls of aluminum foil do not. Is this springiness associated with short range or long-range interactions in paper?
b) Describe the differences between i) statistical segment length, ii) chemical unit length, iii) Kuhn length, and iv) persistence length.
c) Explain how you would apply the idea of a Kuhn unit in linear chains to the crumpled sheet in figure 1 above. (see for example Kantor et al. *Phys. Rev. A* **35** 3056-3071 (1987)).

ANSWERS: 080119 Quiz 2 Polymer Properties

1) a) $z = \alpha\left(\frac{R}{\ell}\right)^{d_f}$

z = mass
R = size
ℓ = step size
α = lacunarity
d_f = mass fractal dimension

b)

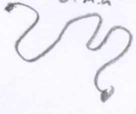
	d_f
rod	1
Gaussian Chain	2
SAW	5/3
Paper	2
Crumpled Paper	~2.5

c) paper & Gaussian Coil

Minimum path, p, & Minimum dimension, d_{min}
connective path, s, or connectivity, c

	d_{min}	c	d_f
Gaussian Chain	2	1	2
Paper	1	2	2

$p^2 \sim z$
$d_{min} = 1$
$c = d_f$



$d_{min} = d_f$; $p = z$

2)

a) $\rho = \frac{z}{R^3} \sim z^{1-3/d_f}$

$R \sim z^{1/d_f}$

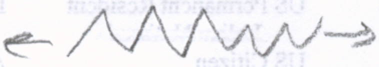
$1 - 3/d_f = -0.7$

$d_f = 1.7$

This doesn't agree since d_f must be larger than 2.

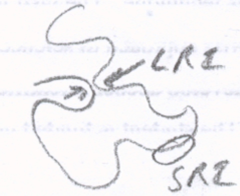
b) $c = 2$

c) Stress in the crumpled sheet is related to how folded the sheet is and will be distributed



along the minimum path.

3) a) SRI is for low chain index differences
LRI is for large chain index differences



b) Springiness is due to SRI

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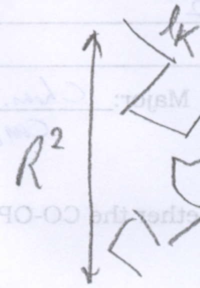
3 c) Kuhn unit is based on freely jointed chain model



$$R^2 = n_K l_K^2$$

For ball of foil consider a slice and find the end to end distance R. Consider that

this 2d projection of the sheet is a random walk similar to the 3-d walk of a polymer chain.

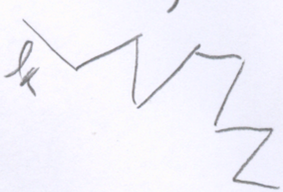
3b)

i) statistical segment length is conversion from chain size R_{eted} to number of chemical units n_{chem}

$$R_{eted}^2 = n_{chem}\, l_{ss}^2$$

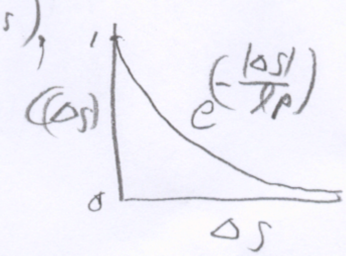
ii) Chemical unit length is just bond length or bond length plus bond angles to compose a chemical unit, l_{chem}.

iii) Kuhn length refers to a freely jointed chain, l_K



$$R^2 = n_K\, l_K^2$$

iv) Persistence length refers to a worm-like chain & describes the decay in orientational correlation $C(\Delta s)$,



$$2 l_p = l_K$$