Quiz 4 Polymer Properties 080206

1) The radius of gyration for an object made of discrete steps, such as a freely jointed chain, can be obtained with summation. For continuous objects, such as disks and rods, an integration over all points must be made to determine R_g.

a) Obtain the radius of gyration for a rod, R_g^2, by performing a normalized double integral over length and radius using a coordinate system with an origin at the center of mass for the rod. You need to use cylindrical coordinates so the integral for a function f(r,y) is $\int_{r=0}^{R}\int_{y=-H}^{H} 2\pi r f(r,y)\,dr\,dy$. (Remember to normalize by an integral with f(r,y) = 1.)

b) Consider a plus sign made of spheres with 3.5 spheres per arm. Calculate the R_g^2 for this object by defining the center of mass, performing a summation and normalizing the result. Does this answer change for a rod of 7.5 spheres per arm (yes or no)? Does it change for a "jack" (6 arm star) (yes or no)?

c) Compare your answer for a plus sign with the radius of gyration for a sphere of diameter 7. (This can be calculated by a normalized spherical integral about the center of mass $\int_{r=0}^{R} r^2\left(r^2 dr\right) \Big/ \int_{r=0}^{R}\left(r^2 dr\right)$ or you can just use the answer and compare if you remember it.)

2) Scattering from a disordered structure such as a polymer coil is often plotted on a log-log scale.

a) Why is a log-log plot used?

b) What equation is used to determine the radius of gyration and sketch the appearance of this function on a log-log plot.

c) In what way is this equation a Gaussian function and why might this be true?

3) Explain how the moiré fringe pattern shown in class relates to scattering by:

a) Explain the conditions for a fringe to occur.

b) Explain the relationship between the fringes and the angle of scattering in terms of spacing.

c) Explain the relationship between the fringes and the angle of scattering in terms of orientation of the fringes (i.e. what angle do the fringes fall on compared to the scattering angle, θ?).

1

1) a) $\int_{r=0}^{R}\int_{-H}^{H} 2\pi r\,(r^2+y^2)\,dr\,dy = 2\pi \int_{-H}^{H}\left[\frac{r^4}{4}+\frac{r^2}{2}y^2\right]_0^R dy$

$= \pi\int_{-H}^{H}\left[\frac{R^4}{2}+\frac{R^2}{2}y^2\right]dy$

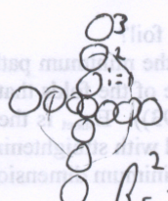
$= \pi\left[\frac{R^4 y}{2}+\frac{R^2 y^3}{3}\right]_{-H}^{H}$

$= \pi\left(R^4 H + \frac{2R^2 H^3}{3}\right)$

$\int_{r=0}^{R}\int_{-H}^{H} 2\pi r\,dr\,dy = 2\pi R^2 H$

$R_g^2 = \frac{\int (r^2+y^2)\,dr\,dy}{\int_V dr\,dy} = \frac{R^2}{2}+\frac{H^2}{3}$

b)



$R_g^2 = \frac{2\left(2\sum^{3} i^2\right)}{N} = \frac{4\cdot(1+4+9)}{13}$

$= \frac{4(14)}{13} = 4.31$

$\boxed{R_g = 2.08}$

For 7 spheres $R_g^2 = \frac{2\left(2\sum_{i=1}^{7} i^2\right)}{N}$

For Jack

$$R_g^2 = \frac{3\left(2\sum_{i=1}^{3} i^2\right)}{1+6(3)} = \frac{6\cdot(14)}{19} = 4.42$$

$$= 2.10$$

Close to the same.

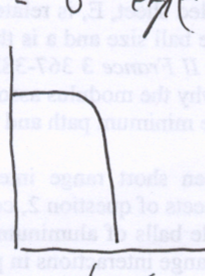
2) a) log-log plot are used to highlight power law functions

$I(q) \sim q^{-d_f}$ for example

b) Guinier's law

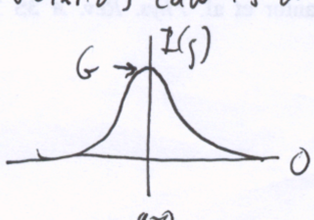
$$I(q) = G\exp\left(-\frac{q^2 R_g^2}{3}\right)$$

log I



log q

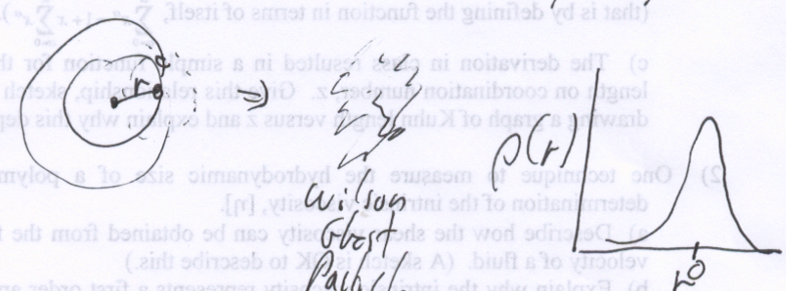
c) Guinier's law is a gaussian distribution about $q=0$



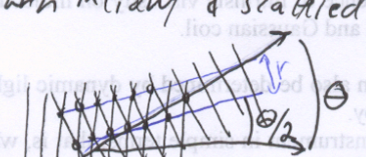
The Fourier Transform of a gaussian is another gaussian so the correlation function is a Gaussian

③

The Gaussian correlation function is related to the Wilson Ghost particle. For any structure we can depict the probability of 2 points separated by a distance "r" being in the particle by rotating the particle about any point in the particle so that a probability density cloud is created whose density distribution function is a Gaussian for any object.



3) a) Fringes occur when incident & scattered waves are in-phase.



b) Fringes occur along a line at θ/2 angle, if the scattered angle is θ. The fringes decrease in spacing with smaller angle & have no upper limit but have a lower limit of λ/2.