Tensile Blob: How Complex Systems Respond to Force

A thermally equilibrated random coil displays an energy that scales with the square of the coil size following the Flory-Krigbaum description discussed in class.

$$\frac{U(R)}{kT} \sim \frac{3}{2}\left(\frac{R}{R_F}\right)^2 \qquad (1)$$

When a force is applied to the coil it deforms according to,

$$F = \frac{dU(R)}{dR} = \frac{3kT}{R_F^2} R \qquad (2)$$

This expression is used to calculate the modulus of an ideal chain, F/R, in rubber elasticity for instance. For weak coil perturbation, $R \sim R_F$, equation (2) can describe a size scale,

$$R_t \sim \frac{kT}{F} \qquad (3)$$

which describes a change in conditions for the coil related to thermal equilibrium and the applied force. For sizes larger than R_t, the modulus of the coil is too weak to sustain the applied force and the chain can stretch. For sizes smaller than R_t, the coil modulus is sufficient to resist the applied force. R_t is called the tensile blob size since it is a size scale chosen by the complex system in response to a tensile force applied to the ends of the structure. The concept of a tensile blob was introduced by Pincus (UCLA) in 1976, *Macromolecules* **9** 386-388 **1976** in somewhat of a different context.

Witten, Rubinstein and Colby, *J. Phys. II France* **3** 367-383 **1993**, broadened the concept of a tensile blob to include other types of complex (multi-size scale) objects, particularly non-thermally equilibrated mass-fractal objects. For this case kT in equation (3) is replaced with an energy associated with the spring constant or stiffness of the object. Nonetheless a scaling relationship between a scaling transition size and the inverse of an applied force is obtained. Witten describes the condition in terms of lever arms with longer distances in the object having a longer lever arm and a weaker response to an applied force, i.e. modulus when compared with smaller lever arms. He then goes on to apply classical mechanics equations to mass-fractal aggregates to determine the structural response to an applied force with several interesting results in regard to reinforced elastomers.

Sukumaran, *Europhysics Journal* **59** 714-720 **2002** used the concept of a tensile blob to describe the structural response of crosslinked chains in a swollen elastomer which was verified by neutron scattering.

The tensile blob is a particularly accessible scaling transition since it can be applied, with some modification, to simple mechanical systems. The tensile blob concept can be easily demonstrated using a crumpled string or sheet of paper. In this case the thermally equilibrated random structure of a polymer coil is replace by the fairly randomly arranged structure that presents two size scales, the overall size and the size of crumples or a persistence length for the string. If a force is applied to the structure it straightens but does so unequally at different size scales. On small scales the crumpling of the structure is at first unperturbed by the external force while on a large scale the structure straightens out either into a line or into a 2D sheet. As more force is applied, the transition size becomes smaller and smaller until finally the smallest scale is straightened.

In class we showed that the scaling behavior of the thermal blob and concentration blob were inverted in that screening leads to Gaussian scaling at large scales for the concentration blob while poor solvent conditions lead to Gaussian scaling at small scales for the thermal blob. In analogy, the tensile blob is inverted relative to the persistence transition. Generally it can be said that external perturbations effect first large scales of a complex structure while internal origin forces effect first local structure.