Thermodynamics
004

1a) The derivation of GTE for an infinite plane of H & W (height and width, respectively) would be mostly similar to derivation of traditional GTE.

$$\Delta T_m(x) = T_{m0} - T_m(x)$$

-5

$$\rightarrow \mu_s(T_m) = \mu_\ell(T_m)$$

$$d\mu = -\bar{S}\,dT + \bar{V}\,dp$$

$$\Rightarrow \cancel{\mu_\ell}(T_m) + \int_{T_m}^{T_m'} -s_\ell\,dT = \cancel{\mu_s}(T_m) + \int_{T_m}^{T_m'} -S_s\,dT + \int_{p_0}^{p} \bar{V}_s\,dp$$

$$\Rightarrow -\int_{T_m}^{T_m'} \underbrace{(\bar{S}_\ell - \bar{S}_s)}_{\Delta_{fus} S}\,dT = \int_{p_0}^{p} \bar{V}_s\,dp$$

$$\Rightarrow -\Delta_{fus} S(T_m' - T_m) = \bar{V}_s \Delta p$$

Typically Δp would be evaluated for $\frac{2\gamma}{R}$, however in this case we will use $\left[\frac{2\gamma_1}{H} + \frac{2\gamma_2}{W}\right]$

$$\text{finally} \Rightarrow \frac{T_m - T_m'}{T_m'} = \frac{2\gamma_1}{H\rho\Delta_m h} + \cancel{\frac{2\gamma_2}{W\rho\Delta_m h}}$$

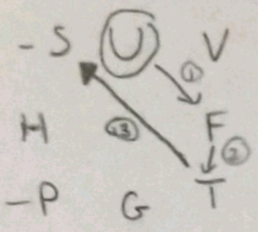
$= (\gamma_L - \gamma_s)$

0

(since width infinite)

the accomodation is to account for the difference in geometry.

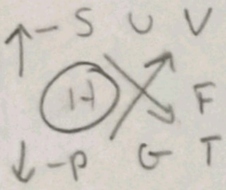
1 b) Solving for U



$$U = F + TS$$

-5

Solving for Ω = $dH = T\,ds + V\,dp$



$$dH = T\,ds + V\,dp$$

Ω is a form of enthalpy

OK

1c)

$$\frac{dP}{dT} = \frac{PL}{T^2 R} \Rightarrow \frac{1}{P}\frac{dP}{dT} = \frac{\Delta H}{RT^2} \Rightarrow \int \frac{d\ln P}{dT} = \int \frac{\Delta H}{RT^2}$$

$$\ln P = \frac{\Delta H}{RT} \cancel{+C}$$ → integrated CCE

$$\ln\left(\frac{P(r)}{P}\right) = \frac{2\gamma V}{k_B T r}$$

$$\Rightarrow \frac{\Delta H}{RT} = \frac{\Delta P\, 2\gamma V}{k_B T r} \Rightarrow \boxed{\Delta T_m = \frac{T_m' \, 2\gamma}{H \rho_s R}}$$

For Ostwald Ripening: $(\mu_i^s)_{r'} - (\mu_i^s)_{r''} = 2V_i^s \gamma^{sl}\left(\frac{1}{r'} - \frac{1}{r''}\right)$

d)

Typical Monte Carlo Algorithm:

$$\boxed{\langle Q \rangle = \frac{1}{M} \sum_{i=1}^{M} Q(z)}$$

-10

e) Homogenous Nucleation

$G_2 = V_s G_v^s + V_L + A_{SL} \gamma_{SL}$

OK

$\Rightarrow G_1 = (V_s + V_L) G_v^L$

$\Delta G^* = G_v^L - G_v^S$

$$\boxed{\Delta G^*_{homo} = \frac{L_v \Delta T}{T_m} = \frac{2 \gamma_{SL}}{r^*}}$$

Heterogenous

$$\Delta G_{het}^* = \Delta G^*_{homo} \, S(\theta)$$

$$S(\theta) = \frac{(2 + \cos\theta)(1 - \cos\theta)^2}{4}$$

θ = wetting angle

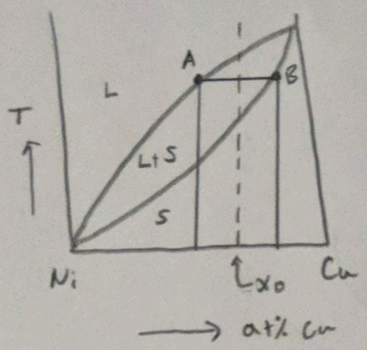
Can only be observed using heterogenous

Thermodynamics

004

2a) The Ni - Cu system is known as isomorphous because of the complete solid solubility of Ni and Cu (meets Hume-Rothery Rules).

OK



Composition of L at dashed line (using the lever Rule)

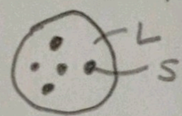
= A composition

Composition of S = B composition

$$\text{amount } S = \frac{x_0 - A}{B - A} \times 100\%$$

$$\text{amount } L = \frac{B - x_0}{B - A} \times 100\%$$

I assume you are reffering to the L + S phase in the phase diagram (white region). The spherical particles would be solid solution forming in the liquid matrix =>



the same method would be used as above to calculate the composition and amounts.

b) Eutectic Reaction => $L \rightleftharpoons \alpha + \beta$

α: Ni β: $ZrNi_5$

Eutectic point = triple point

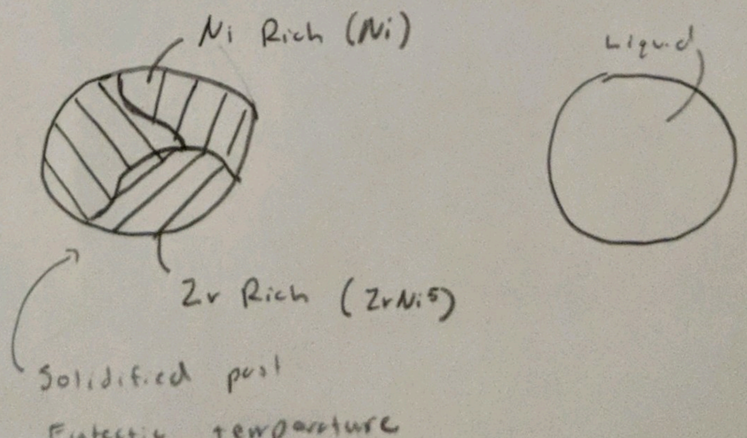
OK

$F = C - P + 1$ ← Pressure constant @ 1 atm

C = components = 2

P = phases = 3

$F = 2 - 3 + 1 = \underline{\underline{0}}$

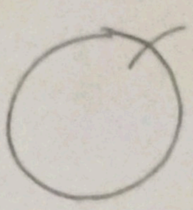


c) Slow Cool | Quench

-5

Temp(°C)

1455



L

100% L

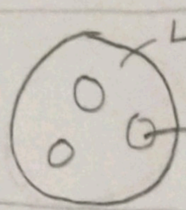
X_0 = Ni 5% Zr

L

100% L

X_0 = Ni 5% Zr

1300



L

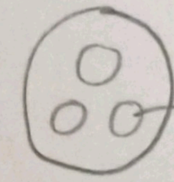
Pro-Eutectic α(Ni)

S = Ni 1% Zr

L : Ni 7% Zr

amount L = $\frac{5-1}{7-1}$ = 66%

amount S = 34%



S = Ni 1% Zr

L = Ni 7% Zr

Pro-Eutectic α(Ni)

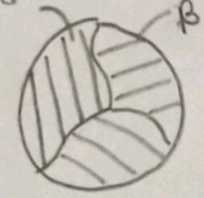
amount L = 66%

amount S = 34%

500 (Bottom of PD)

α (Ni)

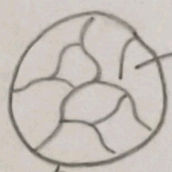
β(ZrNi$_5$)



X_0 = Ni 5% Zr

amount α(Ni) = $\frac{15-5}{15-0}$ = 66%

amount β(ZrNi$_5$) = 34%



α(Ni)

100% α (Ni)

X_0 = Ni 5% Zr

Smaller grain sizes than equilibrium

d) intermetallics are compounds in metal-metal systems with distinct chemical compounds. They are typically identified by vertical lines on the phase diagram and not phase regions. They form due to the stoichiometry being ideal at that composition and forming an ordered phase such as L1$_2$. +2

Ni-Al : Ni_5Al_3, Ni_3Al (2)

Ni-Zr : Zr_2Ni_7, $ZrNi_3$, Zr_8Ni_{21}, Zr_7Ni_{10}, ZrNi (5)

e) Lack of fusion typically occurs due to low energy density:

$$VED = \frac{P}{S \cdot H \cdot L}$$ +10

P= Power

S= Scan speed

H= Hatch Spacing

L= Layer thickness.

where low power, high scan speed, high Hatch spacing, or High layer thickness will eventually lead to irregular lack of fusion porosity. In the charts we see a relativily linear region at the border of the lack of fusion boundary in the process map. The border also mostly follows a single Hatch spacing value. This means that in the linear region the energy density equation can become a linear model: $\frac{P}{S}$. The linear border is a single linear energy density value that transitions from optimal parameters to detrimental lack of fusion. The linearity eventually fails as it transitions into balling (a surface tension and wettability driven defect). Additionally we see that the linear energy density has a threshold where it no longer functions linearly possibly indicting a maximum power that can be proportionally altered with scanning speed to produce Lack of fusion. Keyholing and balling are both observed at higher energy densities. Keyholing is the formation of pores due to entrapped gas and balling is insufficient formation of proper laser tracks. The phase diagrams are important because L-PBF is a rapid solidification process. By understanding the observed and expected microstructure we can understand the solidification process and better Predict the material properties.