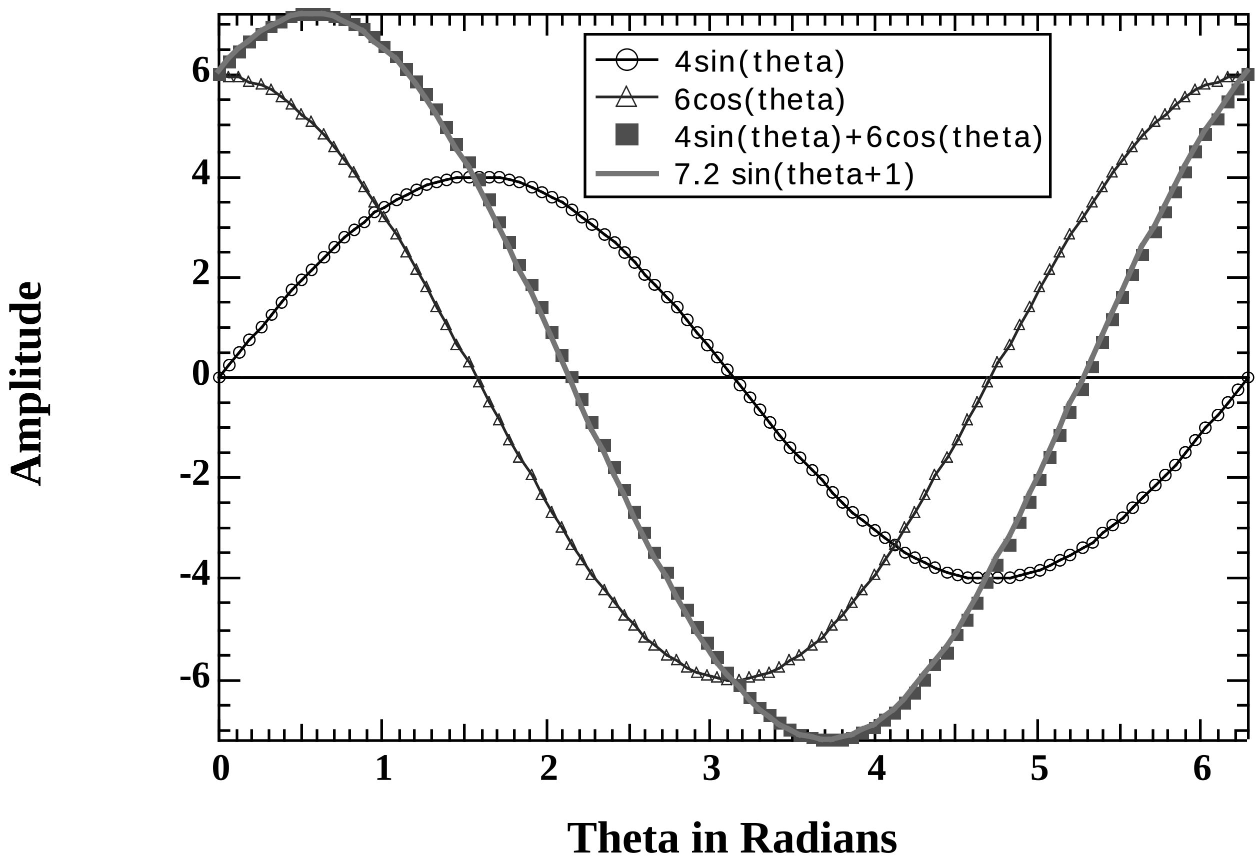
Complex Exponential From Summation of Cosine and Sine Waves:

Any cosine wave with a phase shift of δ can be described by the sum of a sine and a cosine wave of amplitudes A_s and A_c with no phase shift.



$$A \cos(\theta + \delta) = A_c \cos(\theta) + A_s \sin(\theta)$$

This approach was used to plot A_s versus A_c and obtain the phase shift δ.

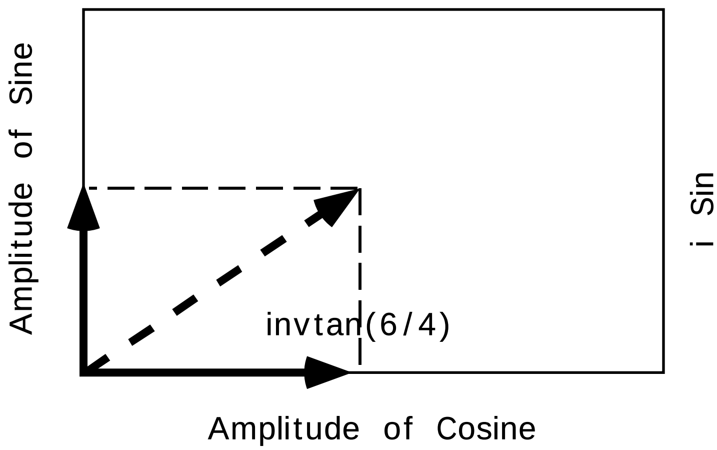


Figure 4-11 on pp. 118 shows this for arbitrary phase waves.

We can use the trigonometry identity,

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

To define A_c and A_s in terms of δ and A:

$$A\cos(\theta + \delta) = \{A\cos(\delta)\}\cos(\theta) + \{-A\sin(\delta)\}\sin(\theta)$$

So, $A_c = \{A\cos(\delta)\}$ and $A_s = \{-A\sin(\delta)\}$.

This presents the problem that A_s is defined as a negative number, yet the amplitude of a wave is required to be a positive number. The issue is resolved by describing the sign of A_s as an imaginary number, i. Then the expression for the phase shifted wave can be written,

$$A(\cos(\delta) + i\sin(\delta)) = e^{i\delta}$$

By expressing phase shifted waves in terms of $e^{i\delta}$ the mathematics for calculation of the diffracted intensity is greatly simplified since a number of simple math identities are available for the complex exponential.

Rule 1: $e^{n\pi i}$ = -1 if n is odd or 1 if n is even, i.e. = $(-1)^n$

Rule 2: $e^{n\pi i} = e^{-n\pi i}$ when n is an integer

Rule 3: $e^{ix} + e^{-ix} = 2\cos x$