Quiz 9 XRD 5/25/01

Power-law decays in small-angle x-ray scattering can be obtained from consideration of the function $I(q = 1/r) = N_p(r)\, n_e^2(r)$.

a) **What is the justification** of the two terms in this equation, i.e. why is the scattered intensity proportional to N_p and why is it proportional to n_e^2? (you can answer this for a collection of particles of size r if it is easier.)

b) For a rod **explain** how this equation can be used to predict $I = B/q$.
What is B for this case?
What are the limits of this power-law scaling?

c) The mass fractal dimension is the power size, r, is raised to in order to obtain mass, i.e. for a disk which is 2-d, Mass = r^2 Thickness, where the thickness is fixed for r > thickness.
For a polymer where $r^2 = nl^2$ what is the mass fractal dimension? (r is the polymer coil size in solution or in a melt, n is the degree of polymerization (mass) and l is the size of a mer unit.)
What power-law decay would you expect for a polymer coil?
Between what limits will this power-law decay be observed for a polymer coil in dilute solution?

d) Most matter will appear as a sharp smooth interface at some size scale.
Explain how $I(q = 1/r) = N_p(r)\, n_e^2(r)$ can be used to obtain Porod's Law, $I(q) = Bq^{-4}$.
Give an expression for B.

e) **Sketch the scattering pattern for a rod** and
Describe what is observed from the large scale to the smallest scale in terms of magnified images of the structure.

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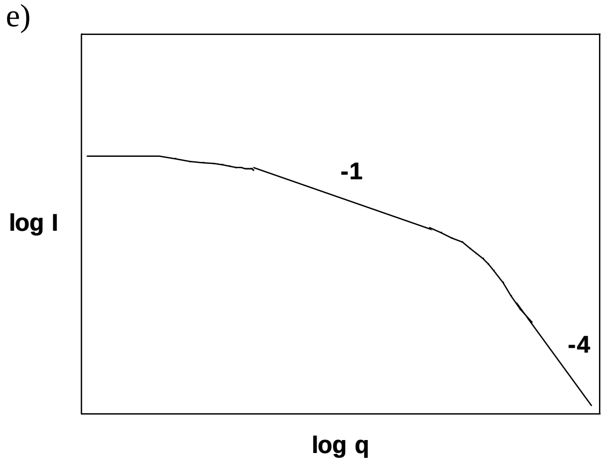
a) The first term just states that for non-interfering sources of scattering the intensity is proportional to the number of sources, N_p. For observation at a size scale r = 1/q of a particle of size r the intensity is proportional to the number of electrons in the particle squared, n_e^2. The justification for this term is the same as the justification for the squaring of the atomic form factor, f, in the calculation of the diffracted intensity. Electrons in the particle are the source of the amplitude observed at or below q = 1/r where r is the particle size. The amplitude is squared to obtain the intensity scattered.

b) For a rod between the length and the diameter the number of particles of size r is equal to L/r. The number of electrons in one of these sub-structures is r R^2, where R is the radius of the rod. Then $I(q=r) = r R^4/L = (R^4/L)q^{-df}$, where d_f=1 for a rod. B is (R^4/L) times structural constants. This power-law decay occurs between L and D.

c) For a polymer Mass = $nM_0 = (r/l)^2M_0$, so d_f = 2. A power-law decay of -2 is expected just like a disk. This would be observed between l and the coil size.

d) The interface region is of thickness of the scale of observation, i.e. r thick. In this interface N_p spheres of size r can be fit where $N_p = S/r^2$. Each sphere contains r^3 electrons so $I = S r^{(6-2)}$, or $I(q) = S q^{-4}$.

e)



At low q the scale of observation is much larger than the rods and the rods appear as specks. The intensity if proportional to the number of specs and the number of electrons per spec (rod) squared. Between the first knee and the second knee the rod is a 1 dimensional object (between the Length and diameter) At smaller scales of observation (higher q) the surface of the rods are observed and the scattering follows Porod's law.