Mathematic Quiz

Solutions

1) All True

2) $|\bar{v}| = v = \sqrt{(2)^2+(3)^2+(-6)^2} = 7$

$\hat{v} = \frac{\bar{v}}{v} = \left(\frac{2}{7}\right)\hat{i} + \left(\frac{3}{7}\right)\hat{j} - \left(\frac{6}{7}\right)\hat{k}$

3) The vector from P to Q is

$\bar{u} = (0-1)\hat{i} + (2-0)\hat{j} + (1-3)\hat{k} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$u = \sqrt{(-1)^2+(2)^2+(-2)^2} = 3$

So the vector from P to Q is given by

$\hat{u} = -\left(\frac{1}{3}\right)\hat{i} + \left(\frac{2}{3}\right)\hat{j} - \left(\frac{2}{3}\right)\hat{k}$

4.) Vectors $\bar{a}$, $\bar{b}$, and $\bar{c}$ are linearly dependent if there exists constants λ, μ, and ν (not all zero) such that $\lambda\bar{a} + \mu\bar{b} + \nu\bar{c} = 0$. Breaking the vectors into components gives

$$\lambda a_x + \mu b_x + \nu c_x = 0$$
$$\lambda a_y + \mu b_y + \nu c_y = 0$$
$$\lambda a_z + \mu b_z + \nu c_z = 0$$

This set has a non zero solution for λ, μ, and ν provided that the determinant of coefficients vanishes, or

$$\begin{vmatrix} a_x & b_x & c_x \\ a_y & b_y & c_y \\ a_z & b_z & c_z \end{vmatrix} = 0$$ which is equivalent to $\bar{a}\cdot\bar{b}\times\bar{c} = 0$

Substituting

$$\begin{vmatrix} 3 & 1 & -2 \\ 4 & -1 & -1 \\ 1 & -2 & 1 \end{vmatrix} = -3 - 1 + 16 - 2 - 4 - 6 = 0$$

So $\bar{u}$, $\bar{v}$, and $\bar{w}$ are linearly dependent and $\bar{v} = \bar{u} + \bar{w}$

Math Quiz - Continued

Solutions

5) First, let $a_{ij} = \cos(x_i', x_j)$

The relative orientation of the individual axes of each system with respect to the other is given by:

	X_1	X_2	X_3
X_1'	a_{11}	a_{12}	a_{13}
X_2'	a_{21}	a_{22}	a_{23}
X_3'	a_{31}	a_{32}	a_{33}

This can be written as a transform tensor as:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

If we define the unit vector along the x_i' direction as $\hat{e}_i'$ then

$$\hat{e}_i' = a_{ij}\,\hat{e}_j$$

So, an arbitrary vector in the unprimed system is given by:

$$\bar{V} = v_j\,\hat{e}_j$$

and in the primed system:

$$\bar{V} = v_i'\,\hat{e}_i'$$

Making the appropriate substitution gives

$$\bar{V} = v_i'\,a_{ij}\,\hat{e}_j$$

The vector components between the two systems are then related by

$$v_j = a_{ij}\,v_i'$$